A Computable Model of Amartya Sen's Social Choice Function in the Framework of the Category Theory Logic

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A significant part of the short history of the mathematical theory of social, political, and economical sciences, specifically in the context of welfare politics and economy, is related with the development of the notion and the theory of the so called "social welfare functions (SWF)". This theory started with the pioneering contributions of A. Bergson and of the Nobel Prize P. Samuelson leading to the so-called "Bergson-Samuelson SWF" [1,2], but received a substantial improvement by the contribution of another Nobel Prize, K. Arrow [3]. Arrow's SWF is intended as a function ranking social states as less, more, or indifferently desirable, for every pair of them, with respect to individual welfare measures and/or preferences.

One of the main uses of SWF is aimed, indeed, at representing coherent patterns (effectively, structures) of collective and social choices/preferences as to alternative social states.

The essential limitation of SWFs is that they are defined in the framework of an approach to the study of social and economic systems stable at equilibrium like in statistical mechanics. They are all inspired, indeed, by Samuelson's general approach to mathematical economics in his seminal handbook [2], based on Gibbs' statistical thermodynamics of gases, to which the first two chapters of the book are significantly dedicated, because naturally consistent with the liberal individualistic vision of economy and society. Unfortunately, a fundamental unexpected and undesired consequence of Arrow's mathematical theory is the famous "Arrow's impossibility theorem", demonstrating the mathematical inconsistency for democratic systems of social choices based on the majority decisions. The fierce debate triggered by Arrow's results — and of which an impressive testimony is the bibliography of the just published second enlarged edition of A. Sen's fundamental book "Collective Choice and Social Welfare" [4] firstly published in 1970 — contributed definitely to the development of the mathematical theory of social choices, as a fundamental mathematical discipline for social, political and economical sciences. The main contribution of A. Sen's theory of social choice functions (SCF) illustrated in this book, for which he was awarded with the Nobel Prize in Economics in 1998, was the formal demonstration that the only way for avoiding Arrow's impossibility results is introducing in the model the interpersonal

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comparison of utilities — and generally the information exchange among persons constituting homogeneous groups, on the contrary considered as irrelevant in the classical economic theory. This allows also to introduce into the mathematical modeling of SCF Theory distributive principles of social and economical justice, such as, for instance, the famous J. Rawl's maxmin principle, which gives priority to the interests of worst-off persons. This possibility transforms SCF Theory into a normative theory of social choices, which takes into account systematically also the ethical and the interpersonal constraints in economy. On this regard, the fundamental contribution of A. Sen was the demonstration that an effective mathematical modeling of ethical constraints in economy cannot be based on abstract and not-computable optimal choices defined on the complete (total) ordering of social/economical states in a society, but on concrete criteria of maximal choices relative to the different contexts, and then defined on partial orderings, not necessarily satisfying a transitive relation among the different social aggregates (sets) of persons so defined, and between groups and the whole society. All this means that the physical paradigm inspiring Sen's mathematical theory of economy and society is no longer the gas thermodynamics stable at equilibrium of the liberalism mathematical model, but the fluid thermodynamics of condensed matter systems, stable in far from equilibrium conditions, characterizing a "liquid society" such as ours. The real-time information exchange among communication agents determines the fast aggregation/dissolution of interest groups in a world-wide environment — think, for instance, at the stock-exchange market and at the infinite flow of data streams it produces. Unfortunately, this condition makes unrealistic a SCF/SWF Theory based on finite [5], and then Turing-computable sets, because on infinite sets Sen's maximal partial orders correspond to as many ultrafilters requiring higher order functions to be calculated [6]. We propose in this contribution an original solution of this problem in the formal framework of the Category Theory, based on the categorical dual equivalence (anti-isomorphism) between coalgebras (environment) and algebras (system), originally applied to the mathematical modelling of condensed matter thermodynamic systems, stable in far from equilibrium conditions, in the framework of quantum field theory of dissipative systems, human brains included [7,8,9]. The same categorical duality coalgebras-algebras is used also in theoretical computer science, for formalizing the effectiveness of dynamic computations on infinite data streams with always changing inner correlations — i.e., on infinite data sets just it is the case of Sen's SCFs. This approach is inside the paradigm of the Algebraic/Coalgebraic Universality in computations [10, 11], which is wider than the classic Turing Universality, also in its versions of the "probabilistic", and then of the "quantum" Turing Machine.

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