Workshop "The origins and evolution of the spacetime"

Pontifical Lateran University - November 27-28, 2018

Roberto Longo

"Noncommutativity, time and entropy bounds"

Abstract.

Classically, one could imagine a completely static space, thus without time. As is known, this picture is unconceivable in quantum physics due to vacuum fluctuations. The fundamental difference between the two frameworks is that classical physics is commutative (simultaneous observables) while quantum physics is intrinsically noncommutative (Heisenberg uncertainty). In this sense, we may say that time is generated by noncommutativity; if this statement is correct, we should be able to derive time out of a noncommutative space.

We know that a von Neumann algebra is a noncommutative space. About 50 years ago the Tomita-Takesaki modular theory showed that there is an intrinsic evolution associated with any given (faithful, normal) state of a von Neumann algebra, so a noncommutative space is intrinsically dynamical! This evolution is characterised by the Kubo-Martin-Schwinger thermal equilibrium condition (Haag, Hugenholtz, Winnink), thus modular time is related to temperature. Indeed, positivity of temperature fixes a quantum-thermodynamical arrow of time.

I shall discuss some aspects of my recent work extending the modular evolution to a quantum operation (completely positive map) level and how this gives a mathematically rigorous understanding of entropy bounds in physics and information theory. A key point is the relation with Jones' index of subfactors. If time allows, I will mention how new, model independent entropy density computations can be made in relativistic quantum field theory by our operator algebraic methods.

References

[1] H. Araki, *Relative entropy of states of von Neumann algebras*, Publ. RIMS Kyoto Univ. 11 (1976), 809-833.

[2] J. Bisognano, E. Wichmann, *On the duality condition for a Hermitean scalar field*, J. Math. Phys. 16 (1975), 985.

[3] O. Bratteli, D. Robinson, "Operator Algebras and quantum statistical mechanics", I & II, Berlin-Heidelberg-New York: Springer Verlag, 1987 & 1997.

[4] A. Connes, *Une classification des facteurs de type III*, Ann. Sci. Ec. Norm. Sup. 6, (1973) 133–252.

[5] S. Doplicher, R. Haag, J.E. Roberts, *Local observables and particle statistics. I*, Comm. Math. Phys. 23 (1971), 199–230.

[6] R. Haag, "Local Quantum Physics – Fields, Particles, Algebras", 2nd edn., Springer, New York (1996).

[7] V. F. R. Jones, *Index for subfactors*, Invent. Math. 72 (1983) 1-25.

[8] R. Longo, *Index of subfactors and statistics of quantum fields. I*, Comm. Math. Phys. 126 (1989), 217-247.

[9] R. Longo, *An analogue of the Kac-Wakimoto formula and black hole conditional entropy*, Comm. Math Phys. 186 (1997), 451-479.

[10] R. Longo, *Notes for a quantum index theorem*, Comm. Math. Phys. 222 (2001), 45-96.

[11] R. Longo, *On Landauer's principle and bound for infinite systems*, Comm. Math. Phys. 363 (2018), 531-560

[12] R. Longo, F. Xu, *Comment on the Bekenstein bound*, J. Geom. Phys. 130 (2018), 113–120.

[13] R. Longo, *Entropy distribution of localised states*, arXiv:1809.03358 [hep-th]

[14] M. Takesaki, "Theory of operator algebras", I & II, Springer-Verlag, New York-Heidelberg, 2002 & 2003.