Dissipation, non-commutative geometry and quantization

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Preamble

- Thermodynamics: system $\to$ (the universe) = (system,bath); open system $\to$ closed system
- Bateman 1931*: damped oscillator $\to$ (damped osc.,amplified osc.)
- Schwinger 1961†: backward & forward in time motion $\mathrm{x(t)} \to (\mathrm{x}_+(\mathrm{t}), \mathrm{x}_-(\mathrm{t}))$
- Wigner function, $\mathrm{x} \to (\mathrm{x} + \mathrm{y}/2, \mathrm{x} - \mathrm{y}/2) \equiv (\mathrm{x}_+, \mathrm{x}_-)$
- Takahashi and Umezawa 1975‡: Thermo Field Dynamics operator field $\mathrm{a} \to (\mathrm{a}, \tilde{\mathrm{a}})$
- GNS (Gelfand-Naimark-Segal) construction in C^*-algebra formalism;
- Hopf algebra, coproduct: $\mathrm{a} \to \Delta\mathrm{a} = \mathrm{a} \times 1 + 1 \times \mathrm{a} = \mathrm{a}_1 + \mathrm{a}_2$

*H. Bateman, Phys. Rev. 38, 815 (1931)

†J. Schwinger, J. Math. Phys. 2, 407 (1961)

‡Y. Takahashi, H. Umezawa, Collective Phenomena 2, 55 (1975); Int.J. Mod. Phys. B 10, 1755 (1996)

Pre-history

Noether theorem:
Lagrangian time-translational invariance $\Rightarrow$ energy conservation
(...closed systems)

all time-dependence through time intervals $(t - t_0)$
Origin of time axis not fixed; all origins are alike

time as a dummy variable

no "after", no "before", no Now, no past, no future, no history,...
no direction in translations, no unidirectional arrow of time,..

Time, in its flowing, swallows those fictitious or false origins one might assign on its axis*, ...as Kronos ate his son...

*G.Vitiello, in *Beyond Peaceful Coexistence - The Emergence of Space, Time and Quantum*, I. Licata Ed., Foreword by G.'t Hooft, Imperial College Press, London 2016, Chap. 17, pp. 465-486.

History

Dissipation modifies this scenario.

Dissipative systems have a lifetime, their “history” can be now recorded, the “memory” of the age of the system is now possible.
The true, unforgettable origin of time is no more arbitrary.
No longer eaten by Kronos...the revenge of his sons...
the *arrow of time* arises.

They “require” the environment; it provides the sink where their waste energy goes, and a source of free energy which feeds them.

The environment is their complement, their Double,
their time-reversed copy.
Time emerges as the observable measured by the Double, which acts as a clock.

Freedom from Unitarily Slavery (freedom from US).

Spontaneous Breakdown of Symmetry may occur.

Phase transitions are possible (unitarily inequivalent dynamical regimes)

Time dependent coherent states are generated.
Trajectories through their manifold are classical, chaotic trajectories.

Dissipative time evolution causes deformations in coherent states which manifest themselves in fractal self-similar forms (morphogenesis).

Diversity appears - The Richness of Babel

A chance to life is given.

Summary

- the doubling of the algebra is related to dissipation and to the gauge structure of the theory,
- the gauge field acts as a reservoir for the matter field
- according to 't Hooft's conjecture, in a regime of completely deterministic dynamics, dissipation appears to play a key rôle in the quantization of the theory*.

⇒ It is argued that the doubling of the algebra carries implicit in its feature the seeds of quantization†.

*G. 't Hooft, Erice lectures (1999); Class. Quant. Grav. (1999); The Cellular Automaton Interpretation of Quantum Mechanics, Springer (2016)

†E. Celeghini, M. Rasetti, and G. Vitiello, Ann. Phys. (N.Y.) 215, 156 (1992)
M. Blasone, P. Jizba, and G. Vitiello, Phys. Lett. A 287, 205 (2001)
M. Sakellariadou, A. Stabile and G. Vitiello, Phys. Rev. D 84, 045026 (2011)
M. Piattelli-Palmarini and G. Vitiello, Biolinguistics 9, 96 (2015)
G. Basti, A. Capolupo, G Vitiello, Progr. Biophys. Molec. Biology 130, 39 (2017)
A. Connes, Noncommutative Geometry (Academic, New York, 1994)

We talk of dissipation and open systems in the same specific sense one observes that in electrodynamics, neither the energy-momentum tensor of the matter field, nor that of the gauge field, are conserved. Indeed,

$$\partial_\mu T^{\mu\nu}_{\text{matter}} = eF^{\mu\nu} j_\mu = -\partial_\mu T^{\mu\nu}_{\text{gauge field}}$$

so that what it is conserved is the *total* $T^{\mu\nu}_{\text{total}}$ **of the** *closed* **system** $\{$**matter field, em field**$\}$

$$T^{\mu\nu}_{\text{total}} = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{gauge field}},$$

i.e. each element of the couple is *open* **(dissipating) on the other one, although the** *closeness* **of the total system is ensured.**

It is in this sense that dissipation enters our discussion of the implications of the algebra doubling.

The algebra doubling and the gauge structure

The “doubling” of the algebra

$\mathcal{A} \to \mathcal{A}_1 \otimes \mathcal{A}_2$ acting on the doubled space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$

is also present in the standard QM formalism of the density matrix and Wigner function.

The doubling of the algebra is implicit even in the classical theory when considering the Brownian motion of a particle and it is related to dissipation.

Consider a simple prototype of *open* systems: the classical damped harmonic oscillator with time independent m, γ and k

$$m\ddot{x} + \gamma\dot{x} + kx = 0\ ,$$

In order to set up the canonical formalism it is required to *close* the system; thus we introduce the time-reversed copy of the x-system in the *doubled* y coordinate*

$$m\ddot{y} - \gamma\dot{y} + ky = 0\ .$$

The $\{x - y\}$ system is thus a closed system. The Lagrangian is:

$$L(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + kxy\ .$$

It is convenient to use the coordinates $x_1(t)$ and $x_2(t)$

$$x_1(t) = \frac{x(t) + y(t)}{\sqrt{2}}\ , \qquad x_2(t) = \frac{x(t) - y(t)}{\sqrt{2}}\ ,$$

*H. Bateman, Phys. Rev. 38, 815 (1931).
E. Celeghini, M. Rasetti, and G. Vitiello, Ann. Phys. (N.Y.) 215, 156 (1992).

$$m\ddot{x}_1 + \gamma\dot{x}_2 + kx_1 = 0 \ ,$$
$$m\ddot{x}_2 + \gamma\dot{x}_1 + kx_2 = 0 \ ,$$

and $\quad p_1 = m\dot{x}_1 + (1/2)\gamma x_2; \qquad p_2 = -m\dot{x}_2 - (1/2)\gamma x_1$.

The Hamiltonian is

$$H = H_1 - H_2 = \frac{1}{2m}(p_1 - \frac{\gamma}{2}x_2)^2 + \frac{k}{2}x_1^2 - \frac{1}{2m}(p_2 + \frac{\gamma}{2}x_1)^2 - \frac{k}{2}x_2^2 \ .$$

We can now introduce the vector potential as*

$$A_i = \frac{B}{2}\epsilon_{ij}x_j \qquad (i,j = 1,2) \ , \qquad (1)$$

with the constant magnetic field $\quad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} = -B\hat{\boldsymbol{3}}$ **and**

$$B \equiv \frac{c}{e}\gamma \quad , \quad \epsilon_{ii} = 0 \quad , \quad \epsilon_{12} = -\epsilon_{21} = 1 \qquad (2)$$

*Y. Tsue, A. Kuriyama, and M. Yamamura, Prog. Theor. Phys. 91, 469 (1994). M. Blasone, E. Graziano, O. K. Pashaev, and G. Vitiello, Ann. Phys. (N.Y.) 252, 115 (1996).

Also, we may write the (oscillator) potential

$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2) \equiv \Phi_1 - \Phi_2$ **with** $\Phi_i \equiv (k/2/e){x_i}^2$

and realize that H_i **(with** $i = 1, 2$**) describe two particles with opposite charges** $e_1 = -e_2 = e$**, namely:**

$$H = H_1 - H_2 = \frac{1}{2m}(p_1 - \frac{e_1}{c}A_1)^2 + e_1\Phi_1 - \frac{1}{2m}(p_2 + \frac{e_2}{c}A_2)^2 + e_2\Phi_2 \ .$$

The Lagrangian can be written in the familiar form

$$
\begin{aligned}
L &= \frac{1}{2m}(m\dot{x}_1 + \frac{e_1}{c}A_1)^2 - \frac{1}{2m}(m\dot{x}_2 + \frac{e_2}{c}A_2)^2 - \frac{e^2}{2mc^2}({A_1}^2 + {A_2}^2) - e\Phi \\
&= \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{c}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi \ .
\end{aligned}
$$

Remarkably, the Lorentzian-like (pseudoeuclidean) metric (the "minus" sign) is not imposed by hand, but required by the doubling of the degrees of freedom. It is crucial in our derivation.

In conclusion, the doubled coordinate, e.g., x_2 **acts as the gauge field component** A_1 **to which the** x_1 **coordinate is coupled, and** ***vice versa*****.**

The energy dissipated by one of the two systems is gained by the other one and viceversa, in analogy to what happens in standard electrodynamics.

The interpretation is recovered of the gauge field as the bath or reservoir in which the system is embedded.

The gauge structure thus appears intrinsic to the doubling procedure.

Similar conclusion is also reached in the case of a fermion field*.

The field algebra is doubled by introducing the fermion tilde-field $\tilde{\psi}(x)$.

The tilde-system is a "copy" (with the same spectrum and couplings) of the ψ-system.

For the massless free Dirac field (extension to the massive fermion case, the boson case and non-Abelian gauge transformation groups is possible) the Lagrangian is

$$\hat{L} = L - \tilde{L} = -\overline{\psi}\gamma^\mu\partial_\mu\psi + \overline{\tilde{\psi}}\gamma^\mu\partial_\mu\tilde{\psi} \ .$$

We assume, for simplicity, that in $\hat{L}$ there is no coupling term of the field $\psi(x)$ with the tilde field $\tilde{\psi}(x)$.

The Hamiltonian for the system is of the form $\hat{H} = H - \tilde{H}$.

*E. Celeghini, E. Graziano, K. Nakamura, and G. Vitiello, Phys. Lett. B 285, 98 (1992); Phys. Lett. B 304, 121 (1993).

One can show that the Lagrangian is invariant under the U(1) local gauge transformation

$$\psi(x) \rightarrow \exp\,[ig\alpha(x)]\psi(x)$$

$$\tilde{\psi}(x) \rightarrow \exp\,[ig\alpha(x)]\tilde{\psi}(x)\ ,$$

since the the "tilde" term in L behaves like the gauge potential term $ig\overline{\psi}\gamma^{\mu}\psi A_{\mu}$ in standard electrodynamics.

Algebra doubling, dissipation and quantization

We now show that, in agreement with 't Hooft's proposal*, loss of information (dissipation) in a regime of completely deterministic dynamics appears to be responsible of the system's quantum mechanical evolution.

Consider the classical damped harmonic $\{x-y\}$-oscillator system and put $x_1 = r \cosh u$, $x_2 = r \sinh u$, $p_1 = \mathcal{C}$ **with** $\mathcal{C} > 0$, $p_2 = J_2$

$$\Gamma = \frac{\gamma}{2m} \, , \; \Omega = \sqrt{\frac{1}{m}(\kappa - \frac{\gamma^2}{4m})} \, , \; \textbf{with} \;\; \kappa > \frac{\gamma^2}{4m} \, .$$

*G. 't Hooft, Classical Quantum Gravity 16, 3263 (1999).
G. 't Hooft, Erice lectures (1999), arXiv:hep-th/0003005.
G. 't Hooft, J. Phys. Conf. Ser. 67, 012015 (2007)
G. 't Hooft, The Cellular Automaton Interpretation of Quantum Mechanics, Springer (2016)

The system's Hamiltonian is then rewritten as*

$$H = \sum_{i=1}^{2} p_i \, f_i(q) \,, \text{ with } \; f_1(q) = 2\Omega, \quad f_2(q) = -2\Gamma.$$

Note that $\{q_i, p_i\} = 1$ and the other Poisson brackets are vanishing.

This Hamiltonian belongs to the class of Hamiltonians considered by 't Hooft.

There, the $f_i(q)$ are nonsingular functions of q_i and the equations for the q's, namely $\dot{q}_i = \{q_i, H\} = f_i(q)$, are decoupled from the conjugate momenta p_i.

A complete set of observables, called beables, then exists, which Poisson commute at all times.

*M. Blasone, P. Jizba, and G. Vitiello, Phys. Lett. A 287, 205 (2001).
M. Blasone, E. Celeghini, P. Jizba, and G. Vitiello, Phys. Lett. A 310, 393 (2003).

The meaning of this is that the system admits a deterministic description even when expressed in terms of operators acting on some functional space of states $|\psi\rangle$, such as the Hilbert space.

We stress that such a description in terms of operators and Hilbert space, does not imply per se quantization of the system: quantization is achieved only as a consequence of dissipation.

We put $H = H_{\mathrm{I}} - H_{\mathrm{II}}$, with

$$H_{\mathrm{I}} = \frac{1}{2\Omega\mathcal{C}}(2\Omega\mathcal{C} - \Gamma J_2)^2, \qquad H_{\mathrm{II}} = \frac{\Gamma^2}{2\Omega\mathcal{C}} J_2^2$$

and impose the constraint

$$J_2|\psi\rangle = 0 \ ,$$

which defines physical states and guaranties that H is bounded from below.

We can then write

$$H|\psi\rangle = H_{\rm I}|\psi\rangle = 2\Omega\mathcal{C}|\psi\rangle = \left(\frac{1}{2m}p_r^2 + \frac{m\Omega^2}{2}r^2\right)|\psi\rangle\,.$$

$H_{\rm I}$ **then reduces to the Hamiltonian for the two-dimensional "isotropic" (or "radial") harmonic oscillator** $\ddot{r} + \Omega^2 r = 0$**.**

The physical states are invariant under time-reversal ($|\psi(t)\rangle = |\psi(-t)\rangle$**) and periodical with period** $\tau = 2\pi/\Omega$**.**

The generic state $|\psi(t)\rangle_H$ **can be written as**

$$|\psi(t)\rangle_H = \hat{T}\left[\exp\left(\frac{i}{\hbar}\int_{t_0}^{t} 2\Gamma J_2 dt'\right)\right]|\psi(t)\rangle_{H_{\rm I}}\ ,$$

where $\hat{T}$ **denotes time-ordering and the constant** $\hbar$**, with dimension of an action, is needed for dimensional reasons.**

The states $|\psi(t)\rangle_H$ and $|\psi(t)\rangle_{H_{\rm I}}$ satisfy the equations:

$$i\hbar\frac{d}{dt}|\psi(t)\rangle_H \;=\; H\,|\psi(t)\rangle_H \;,$$
$$i\hbar\frac{d}{dt}|\psi(t)\rangle_{H_{\rm I}} \;=\; 2\Omega\mathcal{C}|\psi(t)\rangle_{H_{\rm I}}\,.$$

Note: $H_{\rm I} = 2\Omega\mathcal{C}$ **has the spectrum** $\mathcal{H}^n_{\rm I} = \hbar\Omega n$**,** $n = 0, \pm 1, \pm 2, ...$**; since** $\mathcal{C} > 0$**, only positive values of** n **are considered.**

Due to the periodicity of the physical states $|\psi\rangle$ **and using the geometrical phase definition, we obtain**

$$|\psi(\tau)\rangle \;=\; \exp\left(i\phi - \frac{i}{\hbar}\int_0^\tau \langle\psi(t)|H|\psi(t)\rangle dt\right)|\psi(0)\rangle$$
$$\;=\; \exp\left(-i2\pi n\right)|\psi(0)\rangle\,,$$

i.e.,

$$\frac{\langle\psi(\tau)|H|\psi(\tau)\rangle}{\hbar}\tau - \phi = 2\pi n \quad , \quad n = 0, 1, 2, \ldots \;.$$

Using $\tau = 2\pi/\Omega$ and $\phi = \alpha\pi$ leads to

$$\mathcal{H}^n_{\mathrm{I,eff}} \equiv \langle \psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega\left(n + \frac{\alpha}{2}\right) \; .$$

The index n exhibits n-dependence of the state and the corresponding energy. $\mathcal{H}^n_{\mathrm{I,eff}}$ gives the effective nth energy level of the physical system, namely the energy given by $\mathcal{H}^n_{\mathrm{I}}$ corrected by its interaction with the environment.

We conclude that the dissipation term J_2 of the Hamiltonian is responsible for the zero point ($n = 0$) energy: $E_0 = (\hbar/2)\Omega\alpha$.

In QM the zero point energy is formally due to the nonzero commutator of the canonically conjugate q and p operators: the zero point energy *is* the “signature” of quantization.

Our discussion thus shows that dissipation manifests itself as “quantization”.

Using $u(t) = -\Gamma t$, we also have

$$|\psi(t)\rangle_H = \hat{T}\left[\exp\left(i\frac{1}{\hbar}\int_{u(t_0)}^{u(t)} 2J_2 du'\right)\right]|\psi(t)\rangle_{H_I},$$

and

$$-i\hbar\frac{\partial}{\partial u}|\psi(t)\rangle_H = 2J_2|\psi(t)\rangle_H\,.$$

Thus, $2J_2$ induces translations in the u variable: $p_u = -i\hbar(\partial/\partial u)$.

The constraint $J_2|\psi\rangle = 0$, defines families of physical states, representing stable, periodic trajectories and $2J_2$ implements transitions from family to family. We have

$$i\hbar\frac{d}{dt}|\psi(t)\rangle_H = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle_H + i\hbar\frac{du}{dt}\frac{\partial}{\partial u}|\psi(t)\rangle_H\,.$$

The contribution to the energy due to dissipation is thus described by "translations" in the u variable.

Consider the defining relation for temperature in thermodynamics (with $k_B = 1$)

$$\frac{\partial S}{\partial U} = \frac{1}{T}.$$

Using $S \equiv (2J_2/\hbar)$ and $U \equiv 2\Omega\mathcal{C}$, we get $T = \hbar\Gamma$.

Provided S is identified with the entropy, $\hbar\Gamma$ can be regarded as the temperature.

Thus, the "full Hamiltonian" plays the rôle of the free energy $\mathcal{F}$, and $2\Gamma J_2$ represents the heat contribution in H (or $\mathcal{F}$).

Note that the statement that $2J_2/\hbar$ behaves as the entropy is not surprising since it controls the dissipative (thus irreversible loss of information) part of the dynamics.

Noncommutative geometry

We state, without presenting its proof, that an Aharanov–Bohm-type phase interference can always be associated with the noncommutative (X, Y) plane where*

$$[X, Y] = iL^2 \ ;$$

L denotes the geometric length scale in the plane.

*S. Sivasubramanian, Y. Srivastava, G. Vitiello, and A. Widom, Phys. Lett. A 311, 97 (2003).

Consider a particle moving in the plane along two paths, $\mathcal{P}_1$ and $\mathcal{P}_2$, starting and finishing at the same point, in a forward and in a backward direction, respectively.

Let $\mathcal{A}$ denote the resulting area enclosed by the paths. The phase interference ϑ may be written as

$$\vartheta = \frac{\mathcal{A}}{L^2} \; . \tag{3}$$

In the dissipative case, it is convenient to consider the (x_+, x_-) plane (x_+, x_- slightly differ in their definition from the (x_1, x_2) coordinates) and consider the forward and backward in time velocity $v_\pm = \dot{x}_\pm$:

$$v_\pm = \frac{\partial H}{\partial p_\pm} = \pm \frac{1}{m} \left(p_\pm \mp \frac{\gamma}{2} x_\mp \right) \; .$$

They do not commute

$$[v_+, v_-] = i\hbar \frac{\gamma}{m^2} \; .$$

It is thus impossible to fix these velocities v_+ and v_- as being identical.

By putting $mv_\pm = \hbar K_\pm$, a canonical set of conjugate position coordinates (ξ_+, ξ_-) may be defined by $\xi_\pm = \mp L^2 K_\mp$ so that

$$\left[\xi_+, \xi_-\right] = iL^2.$$

This commutation relations characterizes the noncommutative geometry in the plane (x_+, x_-).

We then conclude that, provided $x_+ \neq x_-$, the quantum dissipative phase interference $\vartheta = \mathcal{A}/L^2 = \mathcal{A}\gamma/\hbar$ is associated with the two paths $\mathcal{P}_1$ and $\mathcal{P}_2$ in the noncommutative plane.

Conclusions

- **We have shown that the doubling of the algebra is related to dissipation (in the sense above specified) and to the gauge field structure.**

- **By exploiting 't Hooft's conjecture, according which loss of information within the framework of completely deterministic dynamics, might lead to a quantum evolution, we have shown that the doubling of the algebra carries** *implicit* **in it the seeds of quantization.**

Quantum Field Theory and its Macroscopic Manifestations

Boson Condensation, Ordered Patterns and Topological Defects

Massimo Blasone, Petr Jizba & Giuseppe Vitiello

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