Actualism and Possibilism and
Intensional Possible Worlds

Given our two logistic systems λHST* and HST$^*_\lambda$ as formulations of Russell's and Frege's versions of logical realism, respectively, and HST$^*_\lambda$ as a formulation of conceptual realism, let us now see how either of these systems can provide a deeper account of the distinction between actualism and possibilism and perhaps also the notion of a possible world.

In our third lecture we saw how actualism and possibilism can be distinguished from one another in tense logic and with respect to the logic of the temporal modalities of Aristotle and Diodorus. We could restrict ourselves in our deeper account, accordingly, to second-order tense logic, which would be appropriate for conceptual realism because as forms of conceptual activity, thought and communication are inextricably temporal phenomena.

But a causal or natural modality is also a fundamental part of conceptual natural realism, and with this modality comes the important distinction between what is actual and what is possible in nature, and hence an even more important distinction between actualism and possibilism.

Logical realism does not reject tense logic and the temporal modalities; nor does it reject a causal or natural modality. But it does assume a still stronger modal notion that is not a fundamental part of conceptual realism, namely **a metaphysical modality**.

Now because we do not have clear criteria by which to determine the conditions for a metaphysically possible world, metaphysical necessity and possibility are difficult notions to explicate. What we need is some way to understand the notion of a metaphysically possible world — if such a notion is in fact different from what is possible in nature.

In the ontology of logical atomism we do have a notion of a possible world that is sharp and clear, because every possible world in logical atomism is made up of the same atomic states of affairs, and hence the same simple material objects and properties and relations, as those that make up the actual world, the only difference being between those states of affairs that obtain in a given world and those that do not.

Different possible worlds in logical atomism, in other words, amount to different permutations of being-the-case and not-being-the-case of the same atomic states of affairs, and hence the same simple material objects and properties and relations, that make up the actual world.

As a result, in such an ontology we have a precise notion of what it means to refer to "**all possible worlds**," and dually to "**some possible world**", which are the key notions characterizing how necessity and possibility are understood in any ontology.

Unfortunately, however, the notion of a possible world in logical atomism is not at all suitable for logical realism with its commitment to a realm of abstract objects and a complex network of relations between them.

Question: What notion of a metaphysically possible world is appropriate for logical realism?

We will not attempt to answer this question here, but we will assume that some explication can in principle be given, and that the modal logic for metaphysical necessity contains at least the laws of the modal logic **S5**.

A similar difficulty applies to conceptual realism insofar as one is tempted to introduce a notion of **conceptual necessity** or **possibility**. Here too we have a complex network of concepts and a realm of abstract intensional objects that cannot be accounted for within logical atomism, and for which strictly formal notions of necessity and possibility therefore cannot be given.

A conceptual modality based on the strictly psychological abilities of humans is inadequate to account for these complex networks, and yet how otherwise to formally account for the kinds of model structures that could represent conceptually possible worlds is not all clear.

Question: What notion of a conceptually possible world is appropriate for conceptual (intensional) realism?

We will not attempt to answer this question here as well, but we will assume that the notions of metaphysical necessity and possibility correspond, at least roughly, to the equally difficult notions of **conceptual necessity** and possibility, and hence that the logic of conceptual necessity contains at least the laws of the modal logic **S5**.

Note: We will use $\Box$ and $\Diamond$ as modal operators for both metaphysical and conceptual necessity and possibility. The second-order modal predicate logics with nominalized predicates as abstract singular terms that result from this addition of the laws of **S5** are called $\Box\boldsymbol{\lambda}\mathbf{HST}^*$ and $\mathbf{HST}^*_{\boldsymbol{\lambda}\Box}$.

1. Actualism versus Possibilism in Second-Order Predicate Logic

We assume that the logic of possible objects has been modified in accordance with the previous two lectures, so that it is now a "free logic," just as is the logic of actual objects. The main difference now between the logic of possible objects and that of actual objects is that whereas what is true of every object is therefore also true of every actual object, the converse does not hold. That is, whereas

$$(\forall x)\varphi \to (\forall^e x)\varphi$$

is a basic logical law, the converse is not.

Strictly speaking, our so-called logic of possible objects now deals with more than merely possible objects, i.e., objects that actually exist in some possible world or other. Here, by **existence** — or for emphasis, **actual existence** — we mean **existence as a concrete object**, as opposed to the **being** of an abstract intensional object.

Abstract intensional objects do not exist in this sense even though they have **being** and are now taken as values of the bound object variables along with all possible objects. We can formulate this thesis as follows:

$\neg\mathbf{E}!(\mathbf{Abst})$: $(\forall F^n)\neg E!(F)$,

from which it follows in $S5$ that

$$(\forall F^n)\neg\Diamond E!(F)$$

is provable.

In actualism there is no distinction between being and existence, which means that nominalized predicates must be vacuous, i.e., nondenoting, singular terms. Actualism will validate a thesis such as $\neg\mathbf{E}!(\mathbf{Abst})$, except that it would be expressed in terms of an actualist predicate quantifier such as we describe below.

In possibilism, or rather in what we are now calling possibilism, there is a categorial distinction between being and possible existence, which is expressed in part by the validity of $\neg\mathbf{E}!(\mathbf{Abst})$.

In fact, this particular categorial distinction is one of the many — perhaps indeterminately many — conditions needed to characterize a metaphysically possible world.

For convenience, however, we will continue to speak of the first-order logic of being, which includes possible existence, as simply the logic of possibilism.

Question: How are we to understand the actualist quantifiers when applied to predicate variables?

Thesis: The actualist quantifiers $\forall^e$ and $\exists^e$ when applied to predicate variables refer to **existence-entailing properties, concepts, or relations**, i.e., those properties, concepts or relations that only existing, actual objects **can** have, or fall under, in any given possible world.

The following, which for brevity we state only for properties, are valid theses of the second-order logic of possible objects:

$$(\forall^e F)\varphi \leftrightarrow (\forall F)(\Box(\forall x)[F(x) \rightarrow E!(x)] \rightarrow \varphi),$$

and

$$(\exists^e F)\varphi \leftrightarrow (\exists F)(\Box(\forall x)[F(x) \rightarrow E!(x)] \wedge \varphi).$$

Now many properties, or concepts, and relations are such that necessarily only actual, existing objects can have, or fall under, them. In fact these are the more common properties or concepts that we ordinarily use in our commonsense framework.

Thus, for example, an object cannot be red, or green, or blue, etc., at any time in any possible world unless that object exists in that world at that time.

Similarly an object cannot be a pig or a horse at a time in a world unless it exists at that time in that world; nor, we should add, can there be a winged horse or a pig that flies unless it exists.

Of course, in mythology there is a winged horse, namely Pegasus, and there could as well be a pig in fiction that flies. But that is not at all the same as *actually* being a winged horse or a pig that flies. Indeed, as far as fiction goes, there can even be a story in which there is an impossible object, such as a round square.

Now a fictional or mythological horse is not a real, actually existing horse, and one of the tasks of a formal ontology is to account for the distinction between merely fictional and actually existing objects.

For now it is important to distinguish actual existence from being and merely possible existence.

The two main theses of actualism are:

(1) quantifiers range only over objects that actually exist, and

(2) Predicate quantifiers refer only to those properties, concepts, or relations that entail existence in the above sense, i.e., the only properties, concepts or relations there are according to actualism are those that only actually existing objects can have or fall under.

What this means is that in actualism the quantifiers $\forall^e$ and $\exists^e$, must be taken as primitive symbols when applied to object or predicate variables. The following, of course, is a basic theorem of actualism.

$$(\forall^e F)[F(x) \to E!(x)].$$

In actualism, the statement that *every object exists*, i.e., $(\forall^e x)E!(x)$, is valid, whereas in possibilism the same statement is false.

In fact, given the being of abstract intensional objects, none of which ever exist as actual objects, it is logically false in possibilism that every object exists.

What is true in both possibilism and actualism, on the other hand, is the thesis that *to exist is to possess, or fall under, an existence-entailing property or concept*; that is,

$$E!(x) \leftrightarrow (\exists^e F)F(x)$$

is valid in both actualism and possibilism.

In fact, in possibilism, we can define existence as follows:

$$E! =_{df} [\lambda x(\exists^e F)F(x)].$$

What this definition indicates is that *the concept of existence* is an "impredicative" concept. In other words, because existence is itself an existence-entailing concept — i.e., if a thing exists, then it exists — then, the concept of existence is formed or constructed in terms of a totality to which it belongs.

This is why according to conceptual realism the concept of existence is so different from ordinary existence-entailing concepts, such as being red, or green, a horse, a tree, etc.

The comprehension principle for the actualist quantifiers is not as simple as the comprehension principle (CP^*_λ) for possibilism. It amounts to a kind of *Aussonderungsaxiom* for existence-entailing properties, concepts and relations, ($\mathrm{CP}^{\mathbf{e}}_{\boldsymbol{\lambda}}$).

$(\forall^e G^k)(\exists^e F^n)([\lambda x_1...x_n(G(x_1,...,x_k) \wedge \varphi)] = F)$,

where $k \leq n$, and G^k and F^n are distinct predicate variables that do not occur in φ.

In the monadic case, for example, what this principle states is that although we cannot assume that every open formula φx represents an existence-entailing property or concept, nevertheless conjoining φx with an existence-entailing property or concept $G(x)$ does represent a property or concept that entails existence.

Before concluding this section we should note that although many of our commonsense concepts are existence-entailing concepts, nevertheless there are some, especially relational concepts, that can hold between objects that do not exist in the same period of time or even in the same world.

All existing animals, for example, have ancestors whose lifespans do not overlap with their own, and yet they **are** (now) their ancestors.

An acorn that we choose to crush under our feet will never grow into an oak tree in our world, and yet, as a matter of natural possibility, there is a world very much like ours in which we choose not to crush the same acorn but leave it to grow into an oak tree. It is only a possible, and not an actual, oak tree in our world, but there is still a relation between it and the acorn that we crushed, just as there is an ancestral relation between animals whose lifespans do not overlap.

2. Propositions as Intensional Possible Worlds

The actual world, according to conceptual realism, consists of physical objects and events of all sorts that are structured in terms of the laws of nature and the natural kinds of things there are in the world.

There are intensional objects as well, to be sure, but these have being only as products of the evolution of consciousness, language and culture; and without language and thought they would not have any being at all.

Possible worlds are like the actual world, moreover, i.e., they consist for the most part of physical objects and events structured in terms of the laws of nature.

In speaking of possible worlds here — and even of the actual world — we should be cautious to note that at least in conceptual realism possible worlds are **not** themselves "objects," as when we speak of the various possible kinds of objects and events in the world.

Possible worlds are not values of the bound object variables, in other words, and therefore we do not quantify over them as objects within the formal ontology of conceptual realism.

Now we do seem to quantify over possible worlds in our set-theoretic metalanguage. But, to be precise, what we really quantify over are set-theoretical model structures that we call possible worlds. These set-theoretical model structures are not possible worlds in any literal metaphysical sense; rather, they are abstract mathematical objects that represent the different possible situations described in our formal ontology by means of modal operators.

In logical realism, possible worlds are not really any different than those in conceptual realism, i.e., they are made up of physical objects and events, and, as in conceptual realism, they are **not** themselves *a kind of concrete object* in addition to the possible situations represented by the modal operators.

But unlike the situation in conceptual realism, intensional objects have a mode of being in a separate Platonic realm in logical realism, a realm that is independent of the natural world, and even of whether or not there is a natural world.

This Platonic realm is not only structured logically, as are the intensional objects of conceptual realism, but, because it does not depend upon our cognitive abilities, it goes beyond the latter in having abstract objects that are the intensional counterparts of the possible worlds that are supposedly quantified over in the metalanguage.

That is, even though metaphysically possible worlds are not themselves kinds of objects in the formal ontology of logical realism, nevertheless there are intensional objects in this ontology that are the counterparts of the possible-world models of a set-theoretic metalanguage.

In conceptual realism, on the other hand, there are no such intensional counterparts of possible worlds (models).

This is because, in conceptual realism, but not in logical realism, all intensional objects are ontologically grounded in terms of the human capacity for thought and concept-formation.

This is a fundamental difference between the ontology of logical realism and that of conceptual (intensional) realism.

Now there are at least two kinds of intensional objects in the ontology of logical realism that are counterparts of metaphysically possible worlds (models). For convenience, we will call both kinds **intensional-possible worlds**.

Our first such kind of intensional object is that of a proposition P that is both possible, i.e., $\Diamond P$, and maximal in the sense that for each proposition Q, either P entails Q or P entails $\neg Q$, where by "entailment" we mean necessary material implication, i.e., either $\Box(P \to Q)$ or $\Box(P \to \neg Q)$. Where P is a possible-world counterpart in this sense, we can also read $\Box(P \to Q)$ as 'Q is true in P'.

This notion can be defined by the following λ-abstract:

$$Poss\text{-}Wld_1 =_{df} [\lambda x(\exists P)(x = P \wedge \Diamond P \wedge (\forall Q)[\Box(P \to Q) \vee \Box(P \to \neg Q)])].$$

Note that this λ-abstract is homogeneously stratified. It follows, accordingly, that $Poss\text{-}Wld_1$ stands for a *property, or concept* of both $\Box\lambda\mathrm{HST}^*$ and $\mathrm{HST}^*_{\lambda\Box}$.

In addition, the nominalization of $Poss\text{-}Wld_1$ denotes a value of the bound object variables in both $\Box\lambda\mathrm{HST}^*$ and $\mathrm{HST}^*_{\lambda\Box}$.

Of course, the fact that $Poss\text{-}Wld_1$ is a well-formed predicate that stands for a property or concept does not mean that it must be true of anything, i.e., that there must be propositions that have this property, or fall under this concept.

In fact, any proposition that falls under this concept as an intensional object is so rich in content that it exceeds what is cognitively possible for humans to "grasp" and project as an object of thought: for the actual world it would be *that knowing which everything else is known.*

For this reason the *being*, of such a proposition (as a value of the bound propositional variables) can in no sense be validated in conceptual realism.

But in logical realism, propositions are Platonic entities existing independently of the world and all forms of human cognition. In other words, the "being" of a proposition P is such that $Poss\text{-}Wld_1(P)$ is not constrained in logical realism by what is cognitively possible for humans to have as an object of thought.

Nevertheless, we cannot prove that there are intensional-possible worlds in this sense in either $\Box\lambda\mathrm{HST}^*$ or $\mathrm{HST}^*_{\lambda\Box}$, unless some axiom is added to that effect.

One such axiom would be the following, which posits the "existence," or being, of a proposition corresponding to each metaphysically possible world (model) of the metalanguage.

($\exists$Wld$_1$) $\qquad \Box(\exists P)[Poss\text{-}Wld_1(P) \wedge P].$

One immediate consequence of **($\exists$Wld$_1$)** is that some intensional possible world now obtains, i.e., $(\exists P)[Poss\text{-}Wld_1(P) \wedge P]$ is provable.

We can refer to posited world as "the intensional actual world" — or that knowing which everything else is known.

A criterion of adequacy for this notion of a possible world is that it should yield the type of results we find in the set-theoretic semantics for modal logic. One such result is that a proposition is true, i.e., now obtains, if, and only if, it is true in the actual world. This result is in fact provable on the basis of **($\exists$Wld$_1$)**. That is,

$$Q \leftrightarrow (\exists P)[Poss\text{-}Wld_1(P) \wedge P \wedge \Box(P \to Q)],$$

is provable in both $\Box\lambda\mathrm{HST}^* + (\exists\mathbf{Wld_1})$ and $\mathrm{HST}^*_{\lambda\Box} + (\exists\mathbf{Wld_1})$.

From this another appropriate result follows; namely, that a proposition Q is possible if it true in some possible world; that is,

$$\Diamond Q \leftrightarrow (\exists P)[Poss\text{-}Wld_1(P) \wedge \Box(P \to Q)],$$

is provable in both $\Box\lambda\mathrm{HST}^* + (\exists\mathbf{Wld_1})$ and $\mathrm{HST}^*_{\lambda\Box} + (\exists\mathbf{Wld_1})$.

Another consequence is that Q and Q' are true in all the same possible worlds if, and only if, they are necessarily equivalent; that is,

$$(\forall P)(Poss\text{-}Wld_1(P) \rightarrow$$
$$[\Box(P \rightarrow Q) \leftrightarrow \Box(P \rightarrow Q')]) \leftrightarrow \Box(Q \leftrightarrow Q'),$$

is provable in $\Box\lambda\mathrm{HST}^* + (\exists \mathbf{Wld_1})$ and $\mathrm{HST}^*_{\lambda\Box} + (\exists \mathbf{Wld_1})$.

It does not follow, however, that propositions are *identical* if they are true in all of the same intensional possible worlds.

3. Intensional Possible Worlds as Ways Things Might Have Been

There is yet another notion of an intensional possible world that can be realized in logical realism but not in conceptual realism. This is the notion of an intensional possible world as a property in the sense of "the way things might have been."

David Lewis claimed that possible worlds are "ways things might have been," but for Lewis the "ways that things might have been" are concrete objects and not properties.

Robert Stalnaker noted, however, that "*the way things are* is a property or state of the world, [and] not the world itself," as Lewis would have it. And in general, accordingly, "the ways things might have been" are properties, not concrete objects.

Thus there are possible worlds *qua* properties according to Stalnaker.

But Stalnaker is an actualist, and what this means for him is that except for the concrete actual world all other possible worlds are merely uninstantiated properties. That is, only concrete worlds could be instances of such properties, and for an actualist the only such concrete instance is the actual world. In logical realism, however, the situation is quite different.

In the ontology of logical realism, the "ways things might have been" are properties, but they are not properties of metaphysically possible worlds as concrete objects. Rather, they are properties of propositions; in particular, they are properties of all and only the propositions that are true in the (concrete) possible world in question.

Now a property of all and only the propositions that are true in a given possible world could not be an intensional counterpart of that world if the extension of that property was different in different possible worlds.

Possible worlds are different, in other words, if the propositions true in those worlds are different. What is needed, accordingly, is a property of propositions that does not change its extension from possible world to possible world. We will call such a property a "rigid" property, which we "define" as follows:

$Rigid(F)$: $\quad (\forall x)[\Box F(x) \vee \Box\neg F(x)]$.

Note that because a rigid property will have the same extension in every possible world, the *extension* of that property can then be identified ontologically with the property itself.

The type of intensional-possible world that is now under consideration is that of **a rigid property** (or "**class**") that holds in some metaphysically possible world of all and only the propositions that are true in that world.

This notion can be specified by a homogeneously stratified formula, which means that the property of being an intensional-possible world in this sense can be defined by means of a λ-abstract as follows:

$Poss\text{-}Wld_2 =_{df} [\lambda x(\exists G)(x = G \wedge Rigid(G) \wedge$
$\quad \Diamond(\forall y)[G(y) \leftrightarrow (\exists P)(y = P \wedge P)])].$

Now, as with our first notion of an intensional possible world, the claim that there are possible worlds in this second sense is also not provable in either $\Box\lambda\mathrm{HST}^*$ or $\mathrm{HST}^*_{\lambda\Box}$ unless we add an assumption to that effect. One such assumption is the following, which says that there is such a possible-world property G, i.e., $Poss\text{-}Wld_2(G)$, that holds in any possible world (model) of all and only the propositions that are true in that world:

($\exists$Wld$_2$): $\quad \Box(\exists G)(Poss\text{-}Wld_2(G) \wedge (\forall y)[G(y) \leftrightarrow True(y)]).$

Here by $True(y)$ we mean that y is a proposition that is the case, i.e.,

$$True =_{df} [\lambda y(\exists P)(y = P \wedge P)].$$

Note: the λ-abstract for $True$ is homogeneously stratified, hence specifies a property, or concept, in both $\Box\lambda\mathrm{HST}^*$ and $\mathrm{HST}^*_{\lambda\Box}$.

Now it turns out that we do not have to assume either of the new axioms, $(\exists\mathbf{Wld_1})$ or $(\exists\mathbf{Wld_2})$, to prove that there are intensional possible worlds in the formal ontology of logical realism in either of these two senses.

Both, in fact, can be derived in the modal systems $\Box\lambda\mathrm{HST}^*$ and $\mathrm{HST}^*_{\lambda\Box}$ from what we will call **the principle of rigidity**, **(PR)**, which stipulates that every property, or concept, F, is co-extensive in any metaphysically possible world with a rigid property, i.e., a rigid property that can be taken as the extension of F (in that world). We formulate the principle of rigidity, **(PR),** as follows:

$$\Box(\forall F)(\exists G)(Rigid(G) \wedge (\forall x)[F(x) \leftrightarrow G(x)]).$$

The thesis that there is a rigid property corresponding to any given property is intuitively valid in logical realism where properties have a mode of being that is independent of our ability to conceive or form them as concepts.

In conceptual realism, on the other hand, the thesis amounts to a "reducibility axiom" claiming that for any given concept or relation F we can construct a corresponding *rigid* concept or relation that in effect represents the extension of the concept F. Such a "reducibility axiom" is much too strong a thesis about our abilities for concept-formation.

That $(\exists\mathbf{Wld_2})$ is derivable from **(PR)** follows from the fact that $True$ represents a property in these systems; that is,

$$(\exists G)(Rigid_2(G) \wedge (\forall y)[G(y) \leftrightarrow True(y)])$$

is provable on the basis of (CP^*_λ) in both $\Box\boldsymbol{\lambda}\mathbf{HST}^*$+ **(PR)** and $\mathbf{HST}^*_{\boldsymbol{\lambda}\Box}$+ **(PR)**, and therefore, by the rule of necessitation and obvious theses of **S5** modal logic, it follows that $(\exists\mathbf{Wld_2})$ is derivable from **(PR)**.

A similar argument, which we will not go into here, shows that $(\exists\mathrm{Wld}_1)$ is also derivable in $\Box\boldsymbol{\lambda}\mathbf{HST}^*$+ **(PR)** and $\mathbf{HST}^*_{\boldsymbol{\lambda}\Box}$+ **(PR)**.

The fact that with **(PR)** we can prove in both of the theories of predication $\Box\boldsymbol{\lambda}\mathbf{HST}^*$ and $\mathbf{HST}^*_{\boldsymbol{\lambda}\Box}$ that there are intensional-possible worlds in either the sense of $Poss$-Wld_1 and $Poss$-Wld_2 is significant in more than one respect.

On the one hand it indicates the kind of ontological commitment that logical realism has as a modern form of Platonism. On the other hand, it also indicates a major kind of difference between logical realism and conceptual realism, because, unlike logical realism, the principle of rigidity is not valid in conceptual realism.

What **(PR)** claims about concept-formation is not cognitively realizable for humans. Nor can there be intensional possible worlds in the sense either of $Poss$-Wld_1 or $Poss$-Wld_2 in conceptual realism, because such intensional objects are not cognitively possible for human thought and concept-formation.

Here, with the principle of rigidity and the notion of a proposition as the intensional counterpart of a possible world, we have a clear distinction between logical realism as a modern form of Platonism and conceptual realism as a modern form of conceptualism.

Also, what these differences indicate is that the notions of metaphysical necessity and possibility in logical realism are not the same as the notions of conceptual necessity and possibility in conceptual realism, at least not if conceptual possibility is grounded in what is cognitively realizable in human thought and concept-formation.

Logical realism: The principle of rigidity, **(PR)**, is valid, and therefore so are **($\exists$Wld$_1$)** and **($\exists$Wld$_2$).** That is, there are intensional possible worlds in logical realism in the sense of $Poss$-Wld_1 as well as of $Poss$-Wld_2.

Conceptual realism: The principle of rigidity, **(PR)**, is not valid, and there can be no intensional possible worlds in the sense of either $Poss$-Wld_1 or $Poss$-Wld_2, because such intensional objects exceed what is cognitively possible for human thought and concept-formation.

Therefore, metaphysical necessity and possibility are the not the same as conceptual necessity and possibility.