

# The Cyclic Model Simplified\*

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The Cyclic Model attempts to resolve the homogeneity, isotropy, and flatness problems and generate a nearly scale-invariant spectrum of fluctuations during a period of slow contraction that precedes a bounce to an expanding phase. Here we describe at a conceptual level the recent developments that have greatly simplified our understanding of the contraction phase and the Cyclic Model overall. The answers to many past questions and criticisms are now understood. In particular, we show that the contraction phase has equation of state  $w > 1$  and that contraction with  $w > 1$  has a surprisingly similar properties to inflation with  $w < -1/3$ . At one stroke, this shows how the model is different from inflation and why it may work just as well as inflation in resolving cosmological problems.

## I. THE CYCLIC MANIFESTO

Two years ago, the Cyclic Model [1] was introduced as a radical alternative to the standard big bang/inflationary picture [2]. Its purpose is to offer a new solution to the homogeneity, isotropy, flatness problems and a new mechanism for generating a nearly scale-invariant spectrum of fluctuations.

One might ask why we should consider an alternative when inflation [2–4] has scored so many successes in explaining a wealth of new, highly precise data [5]. There are several reasons. First, seeking an alternative is just plain good science. Science proceeds most rapidly when there are two or more competing ideas. The ideas focus attention on what are the unresolved issues theorists must address and what are the important measurements experimentalists must perform. Inflation has had no serious competition for several years, and the result has been that its flaws have been ignored. Many cosmologists are prepared to declare inflation to be established even though crucial experimental tests remain. Competition stimulates critical thinking and removes complacency.

A second reason to consider an alternative is that, even though inflationary predictions are in marvelous accord with the data thus far, the theoretical front has seen little progress. In fact, if anything, there has been regress. The main questions about inflation that were cited twenty years ago remain today. What is the inflaton and why are its interactions finely-tuned? How did the universe begin and why did it start to inflate?

With the advent of string theory, these issues have become severe problems. Despite heroic efforts to construct stringy inflation models with tens or hundreds of moving parts (fluxes, branes and anti-branes) and examining a complex landscape of (at least)  $10^{500}$  vacua, even a single successful inflationary model is difficult to construct [6]. The notion that there is a landscape of  $10^{500}$  or more string vacua has suggested to some that, if there is an acceptable vacuum somewhere, inflation makes it possible to populate all vacua; and that the ultimate explanation for our universe is anthropic [7]. However, this cannot be the whole story since it begs the question of how the universe started in the first place. No matter where you lie in the landscape, extrapolating back in time brings you to a cosmic singularity in a finite time. The issue of the beginning remains unresolved.

Furthermore, relying on the anthropic principle is like stepping on quicksand. The power of a theory is measured by the ratio of its explanations/predictions to assumptions. A good scientific theory is observationally testable. An anthropic explanation is based upon considerations involving regions of space that are causally disconnected from us and that will, in many cases, never be observed by us. What parameters and properties can vary from region to region? What is the probability distribution? In models such as eternal inflation, the relative likelihood of our being in one region or another is ill-defined since there is no unique time slicing and, therefore, no unique way of assessing the number of regions or their volumes. Brave souls have begun to head down this path, but it seems likely to us to drag a beautiful science towards the darkest depths of metaphysics.

Another unresolved issue is trans-Planckian effects on the production of density perturbations [8]. In inflationary cosmology, the fluctuations observed in the cosmic microwave background had wavelengths at the beginning of inflation that were smaller than the Planck scale. The standard approximation is to assume the initial distribution of sub-horizon and, hence, sub-Planckian fluctuations corresponds to quantum fluctuations on an empty, Minkowski

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background. However, quantum gravity effects may cause the distribution to be different on sub-Planckian wavelengths. The unknown distortion would be inflated and produce an uncertain correction to inflationary predictions for the cosmic microwave background anisotropy.

Finally, the big bang/inflationary picture is still reeling from the recent shock that most of the universe consists of dark energy [9]. The concept had been that, once conditions are set in the early universe, the rest of cosmic evolution is simple. Dark energy has shattered that dream. Dark energy was not anticipated and plays no significant role in the theory. Observations have forced us to add dark energy *ad hoc*.

The current approach in big bang/inflationary model-building has been to treat the key issues – the bang, the creation of homogeneity and density fluctuations, and dark energy – in a modular way. Separate solutions with separate ingredients are sought for each. Perhaps this approach will work, all the problems cited above will be resolved, and a simple picture will emerge. But, perhaps the time has come to consider a different, holistic approach.

The cyclic model has an ambitious manifesto. Its goal is to address the entire history of the universe, past and future, in an efficient, unified approach. There is one essential ingredient – branes in the higher-dimensional picture or a scalar field in the four-dimensional effective theory – that is simultaneously responsible for explaining the big bang; the solution to the homogeneity, isotropy, flatness, and monopole problems [1]; the generation of nearly scale-invariant fluctuations that seed large-scale structure [10, 11]; and, the source of dark energy [1]. Simplicity and parsimony are essential elements. The range of acceptable parameters is broad [12].

Over the past two years, the Cyclic Model has progressed remarkably. The concept has been examined by numerous groups, and many, many useful criticisms and questions have been raised [13–18]. As we and our collaborators have tried to address these issues, the results have been interesting. First, we have discovered that the Cyclic Model already contained the answers. Not a single new ingredient has had to be added thus far. Rather, we have learned to recognize fully the physical properties of the components the model contained at the outset [19–23]. That is, we have been discovering new physical principles stemming from the original model rather than adding new ingredients and patches. Second, as we have come to understand the Cyclic Model better, the picture has become much, much simpler. We believe we can stick by our manifesto: If the model is going to work, it will be because of basic ideas as simple and compelling as inflation. In fact, we find that there are remarkable, unanticipated parallels between inflationary expansion and the contracting and bounce phases of the Cyclic Model [21, 22]. There remain important open issues about the bounce itself, but, now we can confidently say that many of the issues that plagued previous attempts at contracting cosmological models have been cleared away and there are solid reasons for optimism about resolving the remaining issues.

The purpose of this essay is to present the simplified view of the Cyclic Model, focusing on the stages that are most novel and controversial: the contraction and bounce. We focus on the two key ingredients needed to understand the contracting phase: branes and the equation of state  $w > 1$ . As we explain, the two features lead to a series of novel physical effects that solve the homogeneity, isotropy, and flatness problems and ensure a nearly scale-invariant spectrum of density perturbations following the big bang.

## II. THE BASIC CONCEPT

The Cyclic Model was developed based on the three intuitive notions:

- the big bang is not a beginning of time, but rather a transition to an earlier phase of evolution;
- the evolution of the universe is cyclic;
- the key events that shaped the large scale structure of the universe occurred during a phase of slow contraction before the bang, rather than a period of rapid expansion (inflation) after the bang.

The last point means that, unlike previous periodic models, the cycles are tightly interlinked. The events that occurred a cycle ago shape our universe today, and the events occurring today will shape our universe a cycle from now. It is this aspect that transforms the metaphysical notion of cycles into a scientifically testable concept. We can make physical measurements today that determine whether the large scale structure of the universe was set before or after the bang.

The model is motivated by the M-theoretic notion that our universe consists of two branes separated by a microscopic gap (the “bulk”) [24]. Observable particles – quarks, leptons, photons, neutrinos, etc. – lie on one brane and are constrained to move along it. Any particles lying on the other brane can interact gravitationally with particles on our brane, but not through strong or electroweak interactions. So, from our perspective, particles on the other brane are a dark form of matter that cannot be detected in laboratories looking for weakly interacting particles. (The Cyclic Model does not predict whether most of the dark matter detected cosmologically is weakly interacting particles on our brane or particles lying on the other brane. Both are logical possibilities.)

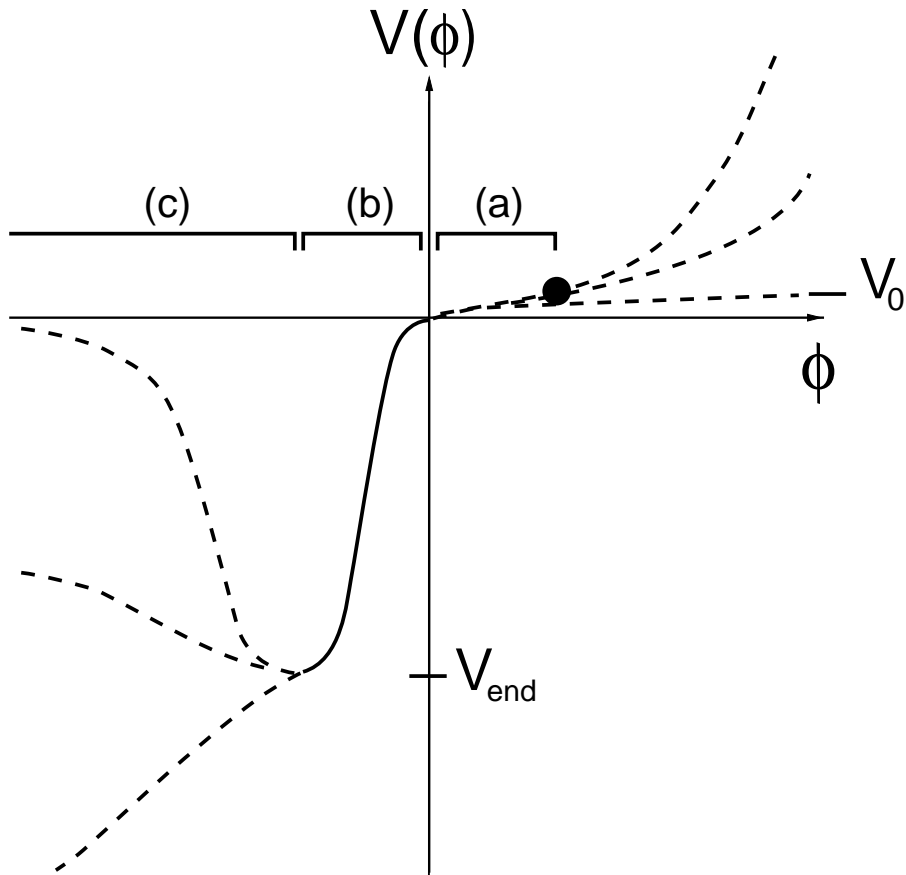


FIG. 1: Scalar potentials suitable for a cyclic universe model. Running forward in cosmic time, Region (a) governs the decay of the vacuum energy, leading to the end of the slow acceleration epoch. Region (b) is the region where scale invariant perturbations are generated. In Region (c), as one approaches the big crunch ( $\phi \rightarrow -\infty$ ), the kinetic energy dominates.

In an exactly supersymmetric vacuum state, the branes do not interact at all. The virtual exchanges of strings and membranes cancel so that there is no force attracting or repelling them. We conjecture that, in a realistic (supersymmetry breaking) vacuum state, an attractive, spring-like force does attract them [1]. Specifically, we imagine that the force is very weak when the branes are thousands of Planck distances apart (as they would be now), so that they are hardly moving. However, the force increases in strength as the branes draw together. Equivalently, we assume an interbrane potential of the form shown in Figure 1, where here  $\phi$  is the moduli field that determines the interbrane separation. When the branes are far apart, the potential is flat and nearly positive; as the branes draw together, the potential falls steeply and becomes negative. When the branes come within a string-scale distance apart (corresponding to  $V \approx V_{end}$  in the Figure), the potential disappears exponentially. Collision corresponds to  $\phi \rightarrow -\infty$ . The scenario can be described by an effective four-dimensional theory for  $\phi$ , where  $\phi$  runs back and forth the potential from some positive value (corresponding to the present brane separation) to  $-\infty$  and back.

The interbrane potential causes the branes to collide at regular intervals. The collision itself is the big bang. The bang is slightly inelastic, infusing the universe with new matter and radiation. From the four-dimensional effective theory, the kinetic energy of  $\phi$  is dominant for a brief period after the bounce, but it decreases rapidly as the universe expands. Hence, after the branes bounce apart, the branes slow down to essentially a halt, and the universe becomes radiation- and matter-dominated. The heat from the collision dominates the universe for a few billion years, but eventually it is diluted enough that the positive interbrane potential energy density dominates. This acts as a source of dark energy that causes the expansion of the branes to accelerate. The matter, radiation, and large scale structure are all diluted away exponentially over the next trillion years or so, and the branes become nearly perfect vacua. However, the interbrane attractive force ensures that the acceleration only lasts a finite time. Inexorably, the branes are drawn together and the potential energy decreases from positive to negative values. The acceleration stops and,

once the potential decreases to the point where  $V = -\frac{1}{2}\dot{\phi}^2$ , the total energy density is zero and the Hubble expansion rate becomes zero. The universe switches from expansion to contraction. The branes themselves do not contract or stretch significantly. Rather, the distance between them shrinks as the two branes crash together. That is, the contraction only occurs in the extra dimension between the branes. The collision is a singularity in the sense that a dimension momentarily disappears. However, the branes exist before, during and after the collision, which plays a crucial role in tracking what happens to the universe through the bounce.

During the dark energy dominated phase, the branes are stretched to the point where they are flat and parallel. During the contraction phase, the branes stop stretching and quantum fluctuations naturally cause the branes to wrinkle. Due to the wrinkles, the branes do not collide everywhere at the same time. Since the collision creates matter and radiation, this means that different regions heat and expand at different times. The result is that the universe is slightly inhomogeneous after the collision. For an exponentially steep interbrane potential, the spectrum of temperature fluctuations is nearly scale-invariant [1, 10, 11].

Unlike cyclic models discussed in the 1920s and 30s, the entropy density does not build up from cycle to cycle. Here is an example of where we take full advantage of the idea of branes and extra dimensions: The entropy created in one cycle is expanded and diluted to near zero density after the dark energy dominated phase, but the entropy density does not increase again in the contraction phase. The simple reason is that the branes themselves do not contract. Only the extra dimensions contract.

From a local observer's point of view, the entropy density undergoes precise cyclic behavior. Yet, the total entropy on the branes grows, in accord with the second law of thermodynamics. It is just that entropy is being exponentially diluted from one cycle to the next, so any given local observer cannot detect the entropy from previous cycles.

The collisions can continue indefinitely despite the fact that the brane collisions are inelastic because gravity supplies extra energy during each contraction phase. During contraction, the kinetic energy of particles or, in this case, branes, is blue shifted due to gravity. This simply means gravity is providing extra kinetic energy in addition to what the interbrane force produces. So, when the branes collide, it is with greater energy than would be obtained with the interbrane force alone. The net result is that gravity adds to the kinetic energy which converts partially to matter and radiation. A key result (shown in Ref. [1]) is that, if we consider the coupled gravity, scalar field, and radiation evolution equations, there exists a cyclic solution that is stable under small perturbations.

### III. PARSIMONY: AN EFFICIENT USE OF SPACE-TIME

The Cyclic Model is more parsimonious than inflation in that a greater proportion of space-time looks like the universe we see. In inflationary models, most of space-time consists either of a very high energy inflating phase, or of the empty vacuum to which bubble interiors tend at late times. With exponential rarity, bubbles are formed in the high energy phase, and, within each, a hot big bang phase forms. The interior of the bubble is hot at first, but the temperature and density decrease steadily with time, and structure formation stops once dark energy dominates the universe. Hence, along any time-like world-line in the inflating universe, there is only a single brief interval (when the world-line crosses a bubble wall) in which there exist stars and galaxies. In the cyclic model, every world-line has repeated, periodically spaced intervals in which stars and galaxies form.

The description of inflation above made the conventional assumption that the interior of a bubble never undergoes further high-energy inflation. If the dark energy is due to a cosmological constant, though, this may not be the case. Imagine a quadratic inflaton potential, say, whose minimum has a small, positive value corresponding to the currently observed dark energy density. High-energy inflation occurs when the inflaton field lies far from the minimum, high up the potential. Inflation ends in a region when the field falls to the minimum. This region is equivalent to a bubble. However, here the minimum corresponds to a low-energy de Sitter phase. With infinitesimally small probability, de Sitter fluctuations can carry the inflaton field back up up the potential high enough to begin a second period of high-energy inflation followed by a second bubble and big bang phase. In this case, a time-like world-line would have irregularly spaced intervals in which stars and galaxies form. However, even in this case, the intervals would be exponentially far apart compared to the model with periodic cycling.

By either reckoning, inflation wastes space-time. In a Bayesian comparison of the two theories, more wasted space-time translates into a reduced probability of a theory being correct. If  $P(A)$  is the probability of theory  $A$  and if  $P(O|A)$  is the probability of observation  $O$  given theory  $A$ , then

$$\frac{P(\text{inflation})}{P(\text{cyclic})} = \frac{P(\text{stars} | \text{inflation})}{P(\text{stars} | \text{cyclic})} \ll 1, \quad (1)$$

assuming equal priors for the two theories. (A similar analysis is sometimes used to explain why inflation is more desirable than the standard big bang model.) We make this point for amusement purposes only. At this point in time,

it seems plausible to assign the models equal priors. However, we hope that future observations and developments in fundamental physics will be the decisive factors.

#### IV. FOREVER CYCLING?

The description in the previous section is an idealization, because there is dissipation from cycle to cycle [25]. For example, black holes formed during one cycle will survive into the next cycle, acting as defects in an otherwise nearly uniform universe. (In the vicinity of the black holes, there is no cycling due to their strong gravitational field.) Also, quantum fluctuations and thermal fluctuations will, with exponentially small rarity, create 'bad regions' which fall out of phase with the average cycling and could form giant black holes [26]. In comoving coordinates, the black holes and bad regions increase in density over time. In this sense, the comoving observer sees the universe as "winding down." Similarly, a local observer will see the cycling as having finite duration in the sense that, at some point, after many, many cycles, he will end up inside a black hole (or bad region) and cease to cycle. Thus, we conclude that cycling conserves energy and is not perfectly efficient; it is neither perpetual motion of the first or second kind. However, because of the stretching of space, the distance between the defective regions remains larger than Hubble distance. New cycling regions of space are being created although any one region of space cycles for a finite time. The cyclic model thereby satisfies the conventional thermodynamic laws even though the cycling continues forever.

It has been suggested that the holographic principle may place a stronger constraint on the duration of cycling [27]. The argument is based on the fact that there is an average positive energy density per cycle. Averaging over many cycles, the cosmology can be viewed as an expanding de Sitter Universe. A de Sitter universe has a finite horizon with a maximal entropy within any observer's causal patch [28] given by the surface area of the horizon. Each bounce produces a finite entropy density or, equivalently, a finite total entropy within an observer's horizon. Hence, the maximal entropy is reached after a finite number of bounces. (Quantitatively, a total entropy of  $10^{90}$  is produced within an observer's horizon each cycle, and the maximum entropy within the horizon is  $10^{120}$ , leading to a limit of  $10^{30}$  bounces.)

Closer examination reveals a flaw in this analysis [25, 29]. Although the overall causal structure of the four-dimensional effective theory may be de Sitter, it is punctuated by bounces in which the scale factor approaches zero. See Figure 2. Each bounce corresponds to a spatially flat caustic surface. All known entropy bounds used in the holographic principle do not apply to surfaces which cross caustics. Hence, holographic bounds can be found for regions of space between a pair of caustics (*i.e.*, within a single cycle), but there is no surface extending across two or more bounces for which a valid entropy bound applies. If the singular bounce is replaced by a non-singular bounce at a small but finite value of the scale factor, the same conclusion holds. In order for a contracting universe to bounce at a finite value of the scale factor, the null energy condition must be violated. However, the known entropy bounds require that the null energy condition be satisfied. Once again, we conclude that the entropy bounds cannot be extended across more than one cycle. Yet another way of approaching the issue is to note that both singular and non-singular bounces have the property that light rays focusing during the contracting phase defocus after the bounce, which violates a key condition required for entropy bounds. In particular, the light-sheet construction used in covariant entropy bounds [30] are restricted to surfaces that are uniformly contracting, whereas the extension of a contracting light-sheet across a bounce turns into a volume with expanding area. Hence, if bounces are physically possible, entropy bounds do not place any restrictions on the number of bounces.

Does this mean that the cycling has no beginning? This issue is not settled at present [31]. We have noted that the cyclic model has the causal structure of an expanding de Sitter space with bounces occurring on flat spatial slices. For de Sitter space, the expanding phase is geodesically incomplete, so the cyclic picture cannot be the whole story. The most likely story is that cycling was preceded by some singular beginning. Consider a universe that settles into cycling beginning from some flat slice in the distant past many bounces ago. Any particles produced before cycling must travel through an exponentially large number of bounces, each of which is a caustic surface with a high density of matter and radiation at rest with respect to the flat spatial slices. Any particle attempting this trip will be scattered or annihilated and its information will be thermalized before reaching a present-day observer. Consequently, the observer is effectively insulated from what preceded the cycling phase, and there are no measurements that can be made to determine how many cycles have taken place. Even though the space is formally geodesically incomplete, it is as if, for all practical purposes, the universe has been cycling forever. We are currently exploring if this picture can be formalized.

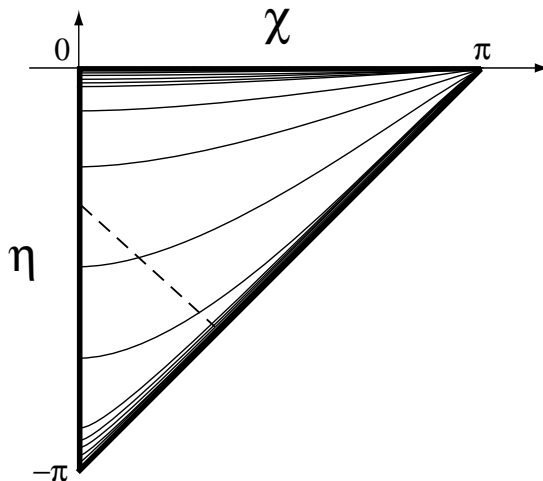


FIG. 2: The cyclic model has an average positive energy density per cycle, so its conformal diagram is similar to an expanding de Sitter space with constant density. The bounces occur along flat slices (curves) that, in this diagram, pile up near the diagonal and upper boundaries. For true de Sitter space, entropy bounds limit the total entropy in the entire spacetime. For the cyclic model, the bounds only limit the entropy between caustics (the bounces). Particles or light-signals emitted in an earlier cycle (or before cycling commences) are likely to be scattered or annihilated as they travel through many intervening cycles (dashed line) to reach a present-day observer. The observer is effectively insulated from what preceded the cycling phase, and there are no measurements to determine how many cycles have taken place.

## V. CONTRACTION AND BOUNCE

Major progress has been made in understanding the most controversial stages of the Cyclic Model: the contraction and bounce. Concerns about these stages are understandable. Previous attempts to construct cyclic or oscillatory models all failed due to various problems that arise during a contraction phase: the matter and radiation density diverge; the entropy density diverges; the 4-curvature diverges; the anisotropy, spatial curvature, and inhomogeneity diverge; collapse exhibits chaotic mixmaster behavior. Hitherto, this pathological behavior has rendered it inconceivable that a nearly homogeneous, isotropic and flat universe with small-amplitude scale-invariant fluctuations could emerge from a bounce.

We now understand that the Cyclic Model can evade these problems because of two distinctive properties:

- (i) *Since matter and radiation are confined to branes, their background densities do not diverge at the bounce. New entropy is created but old entropy remains dilute [1].* Unlike previous cyclic models, the entropy density does not build up from cycle to cycle. Instead, the entropy density returns to near zero towards the end of each cycle.
- (ii) *Because  $w \gg 1$  during the contraction phase, the universe is homogeneous, isotropic and flat [23] with a scale-invariant spectrum of density perturbations [21, 22]. The  $w \gg 1$  condition also ensures that anisotropies are small and first order perturbation theory remains valid until just before the bounce [23].*

These effects due to branes and a  $w > 1$  energy component are novel and critical to the success of a cyclic scenario. Earlier attempts at cyclic models over the last century did not include branes because that concept came into vogue only during the last decade. However, one might naturally wonder why  $w > 1$  was not considered previously. The probable reason is that, prior to inflation, cosmologists often assumed for simplicity that the universe is composed of “perfect fluids” for which  $w = c_s^2$ , where the equation of state  $w$  equals the ratio of pressure  $p$  to energy density  $\rho$ , and the speed of sound  $c_s$  is defined by  $c_s^2 = dp/d\rho$ . If  $w > 1$  and the fluid is perfect, then  $c_s > 1$ , which is physically disallowed for any known fluid. With the advent of inflation, cosmologists have become more sophisticated and flexible about what fluids they are willing to consider. The inflaton, for example, has  $w \approx -1$ , yet the speed of sound is positive and well-behaved. A rolling scalar field with canonical kinetic energy has  $c_s = 1$ . Similarly, it is possible to have  $w > 1$  and yet  $0 \leq c_s^2 \leq 1$  without violating any known laws of physics. This opens the door to a novel kind of cyclic model.

### A. What is the $w > 1$ component?

A  $w > 1$  energy component did not have to be added to the Cyclic Model in order to resolve the traditional problems of contracting universes. The component was there from the very beginning waiting for its effects to be recognized [1, 21]. The  $w > 1$  equation of state is directly due to the interbrane potential that draws the branes together. The interbrane separation is described by a modulus field  $\phi$  with an attractive potential that is positive when the branes are far apart and becomes negative as the branes approach. The rolling from a positive to a negative value is necessary for switching the universe from accelerated expansion to contraction. To see how this occurs, consider the Hubble parameter after the universe is dominated by the scalar field and its potential:

$$H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2} \dot{\phi}^2 + V \right]. \quad (2)$$

The universe is spatially flat after a period of accelerated expansion, so we have included only the scalar field kinetic and potential energy density terms. In order to reverse from expansion to contraction, there must be some time when  $H$  hits zero. Since the scalar field kinetic energy density is positive definite, the only way  $H$  can be zero is if  $V < 0$ . So, reversal from accelerated expansion forces us to have  $V$  roll from a positive value (where  $V$  is the dark energy) to a negative value. However, this automatically creates an equation of state

$$w \equiv \frac{\frac{1}{2} \dot{\phi}^2 - V}{\frac{1}{2} \dot{\phi}^2 + V} > 1 \quad (3)$$

when  $V < 0$  (during the contraction phase). If the potential is exponentially steep,  $\propto -e^{-c\phi}$  with  $c \gg 1$ , it is straightforward to show that it has an attractor solution with equation of state  $w = (c^2 - 3)/3 \gg 1$ . Hence, a  $w > 1$  component is an essential, built-in feature of the Cyclic Model.

### B. $w > 1$ and the homogeneity, isotropy and flatness problems

We first consider the effect of a  $w > 1$  energy component on the average homogeneity, isotropy and flatness of the universe.

As a warm-up, it is useful to recall how inflation homogenizes, isotropizes and flattens the universe. In the standard big bang/inflation model, we imagine that the universe emerges from the big bang with many elements contributing to the Friedmann equation:

$$H^2 = \frac{8\pi G}{3} \left[ \frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\sigma^2}{a^6} + \dots + \rho_I \right] - \frac{k}{a^2}, \quad (4)$$

where  $H$  is the Hubble parameter;  $a$  is the scale factor;  $\rho_{m,r}$  is the matter and radiation density;  $\sigma^2$  measures the anisotropy;  $k$  is the spatial curvature and  $\rho_I$  is the energy density associated with the inflaton. The parameters  $\rho_i$  and  $\sigma$  are constants which characterize the condition when  $a = 1$ , which we can choose without loss of generality to be the beginning of inflation. Each energy density term decreases as  $1/a^{3(1+w)}$ . For the inflaton, we have assumed  $w \approx -1$  and the energy density is nearly  $a$ -independent. The “...” refers to other possible energy components, such as the energy associated with inhomogeneous, spatially varying fields. Inflation works because, as the universe expands, all other contributions, including the spatial curvature and anisotropy, are shrinking quickly as  $a$  grows, while the inflaton density  $\rho_I$  is nearly constant. Once the inflaton dominates, the future evolution is determined by its behavior and its decay products. A complex initial state is focused into a very simple condition after sufficient expansion, a spacetime that is homogeneous, isotropic and spatially flat.

Now let's consider the same equation in a contracting universe. In this case, we do not need an inflaton, but one might imagine a finite cosmological constant instead. In a contracting phase, the cosmological constant becomes rapidly unimportant as the universe contracts and  $a$  shrinks. (Hence, an expanding de Sitter phase is stable to small perturbations, but a contracting one is not.) The term that will naturally dominate is the one that grows the fastest as  $a$  shrinks. In this case, the anisotropy term,  $\sigma^2/a^6$  wins out. A more careful analysis including the full Einstein equation reveals that the universe not only becomes anisotropic, but also develops a large anisotropic spatial curvature and enters a phase of chaotic mixmaster behavior as the bounce approaches. The anisotropy causes the universe to stretch in one direction while collapsing in other directions, forcing the geometry to become cigar-like. Next, the anisotropic spatial curvature grows rapidly, forcing the anisotropic stretching to stop and switch direction. When the anisotropic stretching makes the universe cigar-shaped again, the spatial curvature term once again causes the anisotropic stretching and another switch in direction. The evolution follows a chaotic path in which the bounces

from one stretch direction to another appears to follow a random pattern. This behavior is called “chaotic mixmaster behavior” [32]. The mixmaster behavior produces severe inhomogeneities. The anisotropic contraction follows a set of dynamical equations with chaotic solutions. Hence, the directions of stretching and contracting seem to follow a random pattern that is exponentially sensitive to initial conditions. As a result, if the universe is even slightly inhomogeneous when the contraction phase begins, it will become increasingly inhomogeneous as different regions undergo different chaotic mixmaster collapse [33]. The universe approaches the singularity in a state that is wildly anisotropic, curved and inhomogeneous – a cosmological disaster.

However, the story changes completely when we add an energy density component with  $w > 1$  [23]. The brane/scalar field kinetic energy density decreases as

$$\frac{\rho_\phi}{a^{3(1+w)}}, \quad (5)$$

where the exponent  $3(1+w) > 6$ . Now, the scalar field density grows faster than the anisotropy or any other terms as the universe contracts. The longer the universe contracts, the more the scalar density dominates so that, by the bounce, the anisotropy and spatial curvature are completely negligible. Also negligible are spatial gradients of fields. The evolution is described by purely time-dependent factors, a situation referred to as *ultralocal*.

In short, the striking discovery is that *a contracting universe with  $w > 1$  has the same effect in homogenizing, isotropizing and flattening the universe as an expanding universe with  $w < -1/3$* . This realization enables us to address the comment by some that the Cyclic Model is actually a model of inflation since it includes a period of dark energy domination that helps to homogenize and flatten the universe for the next cycle [14]. (Dark energy can be viewed as a form of ultra-slow inflation.)

Now we see that this characterization is incorrect. As proof, consider an alternative model in which the period of dark energy expansion is followed by a period of contraction with  $w < 1$ . Despite being rather homogeneous and flat at the end of the dark energy dominated period, the universe would be highly inhomogeneous, anisotropic and randomly curved at the bounce. The universe would, therefore, emerge in an unacceptable state and would not satisfy the conditions for cycling.

In short, the key element for making the universe homogeneous, isotropic and flat, for avoiding mixmaster behavior, and for insuring a cyclic evolution is the contraction phase with  $w > 1$ .

### C. $w > 1$ and scale-invariant perturbations

The fact that  $w$  is greater than one during the contraction phase is not only critical for the background homogeneous solution, but also for the perturbations [21, 22]. Both the Cyclic Model and inflation generate density perturbations from sub-horizon scale quantum fluctuations. In each picture, there is one phase when the sub-horizon scale fluctuations exit the horizon and a much later phase when they re-enter.

In both cases, fluctuations leave the horizon because of the value of  $w$ , or, equivalently,  $\epsilon \equiv \frac{3}{2}(1+w)$ . For a universe with constant  $\epsilon$ , the scale factor  $a(t)$  and the Hubble radius  $H^{-1}$  are related by the Friedmann equations

$$a(t) \sim t^{1/\epsilon} \sim (H^{-1})^{1/\epsilon}. \quad (6)$$

If the universe is expanding and quantum fluctuations are supposed to leave the horizon, then it is necessary that  $a$  grows faster than  $H^{-1}$ . From the relation above, this requires  $\epsilon < 1$  (or  $w < -1/3$ ). To obtain a nearly scale-invariant spectrum of fluctuations, it is necessary that  $H^{-1}$  change very little during a period long enough for many modes to be stretched beyond the horizon. This occurs in the limit  $\epsilon \ll 1$ . In fact, the scalar spectral index  $n_s$  about some given wavenumber can be computed by standard methods to be [21]

$$n_s - 1 = -2\epsilon + \frac{d \ln \epsilon}{dN} \quad (7)$$

where  $N$  is a time-like variable that measures the number of e-folds of inflation remaining when a given mode exits the horizon. Here we see that, indeed, if  $\epsilon$  is small and nearly constant, a nearly scale-invariant spectrum is predicted.

Now consider a contracting universe. Both the wavelength of the fluctuations and the Hubble horizon are shrinking. In order for a mode to exit the horizon, the horizon must shrink faster than the wavelength of the mode, or, equivalently,  $H^{-1}$  must decrease *more rapidly* than  $a(t)$ . According to (6), this requires  $\epsilon > 1$  or  $w > -1/3$ . To obtain a spectrum that is nearly scale-invariant, we need  $a$  to be nearly constant over a period when  $H^{-1}$  changes a lot. This occurs if  $\epsilon \gg 1$  or  $w \gg 1$ . This is precisely what is obtained during the contraction phase of the Cyclic Model if the interbrane potential is negative and exponentially steep. Using standard methods, we obtain for the spectral index [21]

$$n_s - 1 = -\frac{2}{\epsilon} - \frac{d \ln \epsilon}{dN} \quad (8)$$



which, consistent with our intuitive analysis, predicts a nearly scale-invariant spectrum for  $\epsilon \gg 1$  and nearly constant.

Comparing (7) and (8), we see that the two expressions are related by the transformation  $\epsilon \rightarrow 1/\epsilon$ . That is, the perturbations produced in the contracting phase are dual to the perturbations in the expanding phase. Recently, L. Boyle *et al.* [22] have shown that this surprising duality extends for all  $\epsilon$ . That is, *for each inflating model with a scalar spectral index  $n_s$ , there is a corresponding contracting (ekpyrotic) model with the same  $n_s$ .*)

A corollary is that the Cyclic model and inflation cannot be distinguished by observing the (linear) scalar perturbations alone. We must dig further to determine if the perturbations were produced in a rapidly expanding phase or a slowly contracting phase.

One approach is to measure the spectral index of tensor perturbations [1, 34]. Here the two models make starkly different predictions. Inflation predicts a nearly scale-invariant spectrum ( $n_s \approx 0$ ) and the Cyclic Model predicts a very blue spectrum  $n_s \approx 2$ . The difference arises because gravity plays a different role in the two scenarios.

In inflation, gravity plays a dominant role in expanding the universe and in creating a nearly de Sitter background that excites all light degrees of freedom. Since the inflaton is nearly massless and the tensor modes are precisely massless and since they both evolve in the same (nearly) de Sitter background, both obtain a (nearly) scale-invariant spectrum. Furthermore, unless some additional parameters, fields and/or tunings are introduced in the inflation model, the amplitude of the spectra are comparable.

In the Cyclic Model, the situation is entirely different. The universe is so slowly contracting that gravity is nearly irrelevant. That is, the gravitational background is nearly Minkowski. The scale-invariant fluctuations arise because one field,  $\phi$ , has a very steep potential. As  $\phi$  rolls down a steep potential, quantum fluctuations are unstable and amplified. Since  $\phi$  determines the distance between branes, this means that the fluctuations in the time of collision grow. For a negative, exponentially decreasing potential, one can show that  $w \gg 1$  and, hence, the spectrum of  $\phi$  fluctuations is nearly scale-invariant (see discussion in Sec. V A). However, only  $\phi$  obtains scale-invariant fluctuations since it is the only field with the steep potential. All other light fields, including the tensor modes, only see a slowly contracting (nearly Minkowski) gravitational background. This background produces a blue spectrum.

The cosmological background in the Cyclic Model is not only slowly contracting, but the value of the Hubble parameter is exponentially small compared to inflation. As a result, higher order non-gaussian corrections to the density perturbation spectrum, which are proportional to  $H/M_{pl}$ , are exponentially suppressed compared to inflation. Also, since  $\phi$  is the only field to obtain scale-invariant fluctuations, it is not possible, for all practical purposes, to construct examples where perturbations in other degrees of freedom compete in producing the observed fluctuation spectrum. This is why the perturbation spectrum predicted by the Cyclic Model is *super-gaussian* and *super-adiabatic* compared to inflation.

Finally, we recall that there is an uncertain contribution to the inflationary prediction of density perturbations due to trans-Planckian effects [8]. That is, the modes observed on the horizon today have wavelengths at the beginning of inflation that are smaller than the Planck scale (in most inflationary models). The conventional calculations of the inflationary density perturbation spectrum assume that the initial fluctuations on small scales corresponds to ground state fluctuations in empty, flat Minkowski space. This approximation may be invalidated for sub-Planckian fluctuations due to quantum gravity effects. The magnitude of the correction is debatable. It is worth noting that sub-Planckian effects should not be a factor in cyclic models. This is because the perturbation modes do not escape the horizon by having their wavelengths stretched. Rather, their wavelengths remain nearly constant while the horizon shrinks. The wavelengths of fluctuations observed in the cosmic microwave background escaped the horizon during contracting when the Hubble radius was exponentially larger than the Planck scale, and so sub-Planckian effects should be irrelevant. Hence, the trans-Planckian issue is avoided in cyclic models and the prediction of density perturbations is more certain than inflation, where the introduction of additional fields and trans-Planckian physics can change nearly all of the predictions of the simplest inflationary models.

#### D. $w > 1$ and the bounce

Because  $w > 1$  during the contraction phase, the conditions leading up to the bounce are ultralocal [23]. Ultralocality means that the evolution equations are, to an excellent approximation, only dependent on time-varying quantities. Spatial variations can be ignored. One way to view the situation is that the Hubble horizon is contracting around any observer in the contracting spacetime, so differences between distant points are irrelevant to the local evolution. There is simply not time before the bounce for distant regions to interact. Hence, instabilities are suppressed and evolution is smooth. This simplicity is essential for analyzing the propagation of perturbations through the bounce.

The subject has been controversial in large part due to a subtle difference in the nature of growing and decaying modes in a contracting phase compared to an expanding phase. In inflation, the curvature fluctuation on comoving hypersurfaces is a familiar growing mode, and the time delay is a decaying mode; but, the roles are reversed in

a contracting phase. Hence, the curvature fluctuation on comoving hypersurfaces shrinks to zero as the bounce approaches.

One of the theorems learned from studying perturbations in inflation is that the curvature fluctuation is conserved for modes outside the horizon [3]. If this remained true for the Cyclic Model, then zero curvature fluctuation before the bounce implies zero curvature fluctuation after the bounce. A corollary is that the growing time-delay mode during contraction transforms entirely into a decaying mode after the bounce, and there is no scale-invariant spectrum of curvature perturbations in the expanding phase. This would be correct if the bounce occurs on a comoving hypersurface so that the curvature perturbation can be matched continuously.

However, in four dimensions, the bounce is singular in all dimensions, and the choice of matching hypersurface at the bounce is unclear. Some have nevertheless argued that a comoving hypersurface is the only plausible choice [15]. They conclude that no curvature fluctuations exist after the bounce and the model fails. Others choose a different hypersurface for the bounce, in which case the curvature fluctuation is not continuous at the bounce [10]. For any choice of matching surface *other than* the comoving hypersurface, the growing, time-delay mode during contraction transforms into a mixture of decaying (time-delay) mode and growing (curvature fluctuation) mode after the bounce [16]. The curvature fluctuation, thereby, inherits a fraction of the initial time-delay mode and produces an observationally acceptable, nearly-scale invariant spectrum of fluctuations in the expanding phase. Others claim that the matching hypersurface is ambiguous [16, 17], and so the Cyclic Model makes no definite prediction for the density perturbation amplitude.

In the Cyclic Model, though, the four-dimensional picture is only an approximation describing a higher dimensional picture of colliding branes. The branes make the critical difference [11, 35, 36]. The branes exist before, during and after the bounce, and they fix a precise hypersurface for the bounce. Namely, the bounce corresponds to a time-slice in which each point on one brane is in contact with a point on the other brane. That is, the appropriate matching hypersurface is the one in which the bounce is everywhere simultaneous [11, 25]. Since  $\phi$  is the modulus field that determines the bounce between branes, one might imagine that this corresponds to  $\delta\phi = 0$ , which is a comoving hypersurface. However,  $\phi$  only measures the distance between branes if they are static. If the branes are moving, there are corrections to the distance relation that depend on the brane speed and the bulk curvature scale [11]. Roughly speaking, the condition that  $\delta\phi = 0$  ensures no velocity perturbations tangent to the branes, but simultaneous bounce requires no relative velocity perturbations perpendicular to the branes. The gauge transformation required to transform from  $\delta\phi = 0$  to the proper gauge introduces a scale-invariant curvature perturbation on the surface of collision. Thus, in a collision-simultaneous gauge, the spatial metrics on the branes *necessarily* acquire long wavelength, nearly scale invariant curvature perturbations at the collision. This result has been obtained using several different methods by three independent groups [11, 35, 36].

The result for the long wavelength curvature perturbation amplitude in the four-dimensional effective theory, propagated into the hot big bang after the brane collision is [11, 12]:

$$\zeta_M = \frac{9\epsilon_0}{16k^2L^2} \frac{\tanh(\theta/2)}{\cosh^2(\theta/2)} (\theta - \sinh \theta) \approx -\frac{3\epsilon_0 V_{coll}^4}{64k^2L^2} \quad (9)$$

where  $\theta$  is the rapidity corresponding to the relative speed  $V_{coll}$  of the branes at collision, and the second formula assumes  $V_{coll}$  is small.  $L$  is the bulk curvature scale, and  $\frac{\epsilon_0}{16k^2}$  has a scale invariant power spectrum. The presence of radiation on the branes before or after the collision produces an additional correction term given in full in Ref. VII. As discussed above, the physical origin of the curvature perturbation is in the time delay between the collision timeslice and the  $\delta\varphi = 0$  (or comoving) hypersurfaces, on which the branes do not possess long wavelength scale invariant curvature perturbations.

We emphasize that the key to this result is the branes – a new, unanticipated aspect of branes that turns out to be essential to the Cyclic Model. The orbifolding of the extra dimensions means that the extra dimension is non-uniform. In particular, as the branes approach, we found that bulk excitations perturb the collision-simultaneous time-slice from comoving. For comparison, if we consider an extra dimension that is Kaluza-Klein compactified, we reach a different conclusion. Then, the comoving slicing is well-defined up to and including the bounce and no scale-invariant perturbations propagate into the expanding phase.

## VI. ONWARD TO THE NON-LINEAR REGIME

Thus far, we have analyzed the propagation of perturbations through the bounce assuming they are linear. From this, we have learned that branes introduce a new physical element essential for propagating perturbations through the bounce. Also, we have obtained, we hope, a good estimate of the spectral amplitude. However, a full analysis including the non-linear physics close to the bounce is required to complete the picture.

Because  $w > 1$  during the contraction phase, the universe remains nearly homogeneous, isotropic and flat and the linear approximation remains valid up until the branes are about a string scale-length apart. At this point, corrections to the Einstein action become important. We cannot be sure what those corrections are. However, assuming there is a bounce, they operate for a very short time. The branes lie within a string-scale-length for roughly a string scale-time, or about  $10^{-40}$  seconds.

During these last instants before the bounce, the modes of interest for cosmology, *e.g.*, wavelengths which lead to the formation of galaxies and larger scale structure, are far outside the horizon and their dynamics is frozen. Although their amplitude may become non-linear, there is not enough time during the bounce for interactions to alter the long-range correlations. Hence, we conjecture, it is reasonable to match the linear behavior just before the bounce to the linear behavior just after the bounce.

In fact, one approach to the matching problem may be to avoid  $t = 0$  altogether by analytically continuing in the complex  $t$ -plane in a semicircle with radius greater than the string scale and connecting negative to positive real values of  $t$  [25]. Then, the linear analysis described above would remain essentially uncorrected by nonlinear gravitational effects, at least on long (three-dimensional) wavelengths. Work is currently in progress to construct such a continuation in nonlinear gravity.

On the other hand, for modes with wavelength less than  $10^{-30}$  cm, causal dynamics can alter the dynamics in the final instants. In particular, the non-linear corrections to gravity could conceivably produce large amplitude effects that lead to the formation of many tiny primordial black holes.

The black holes are bad news for those wishing to track precisely what occurs at the bounce. String theoretic methods are probably not powerful enough to analyze precisely this kind of inhomogeneous, non-linear regime. However, from a cosmological point-of-view, it is straightforward to envisage their effect, assuming that the branes bounce.

The black holes are small and have a mass roughly of order the mass density times Hubble volume at the collision. This scale is model-dependent, but for the wide range of parameters allowed for the cyclic model based on other constraints [12], the mass is sufficiently small that the black holes decay in much less than one second, well before primordial nucleosynthesis.

We conjecture that the black holes can be a boon to the scenario. (See also [37], who consider an alternative model that begins with a dense gas of black holes.) Their lifetime is long enough that they likely dominate the energy density before they decay. Consequently, their evaporation provides the entropy observed today. When they decay, their temperature rises near the end to values high enough to produce massive particles with baryon-number violating decays. At this point, the black holes are much hotter than the average temperature of the universe, so the decays occur when the universe is far from equilibrium. Assuming CP-violating interactions also exist, as in conventional high-temperature baryogenesis scenarios, the decay can produce the observed baryon asymmetry [38]. In addition, the decay can produce dark matter particles that can meet current observational constraints.

## VII. WHAT WE HAVE LEARNED

Our study of the Cyclic Model has uncovered surprising new facts about contracting universes. Namely, a contracting universe with  $w > 1$  has remarkable properties analogous to an expanding phase with  $w \leq -1/3$ . The homogeneous solution to the Friedmann equation becomes spatially uniform, isotropic and flat. At the perturbative level, a nearly scale-invariant spectrum of density perturbations is generated. There is a precise duality relating (linear) scalar perturbations produced in an inflating phase to those produced in a contracting (ekpyrotic) phase. The evolution equations are also ultralocal (purely time-dependent) at least up until stringy corrections to the Einstein equation become significant when the branes are separated by less than a string scale-length.

Consequently, many of the conventional worries of the past about contracting phases are addressed, and attention is turning to what happens in the final instants before and after the collision. The goal is to determine if: (a) the bounce occurs; and, (b) perturbations on wavelengths greater than the string scale lengths (which includes the wavelengths of cosmological interest) obey the matching rule naturally inferred by analytically continuing the linear solution. Current research is focused on these exploring these issues.

The outcome has profound implications for cosmology and fundamental physics. If inflation is correct, then we are blocked from any direct knowledge of the big bang and any other pre-inflationary conditions by a period of superluminal expansion. If the Cyclic Model is correct, then our measurements of microwave background fluctuations and large-scale structure are *to leading order* direct probes of the big crunch and big bang, including stringy and extra-dimensional physics, as illustrated by Eq. (9). Settling the cosmological issue of whether the density fluctuations were produced during a period of expansion or contraction (by searching for tensor fluctuations, non-gaussianity and non-adiabaticity) will also determine whether physical conditions near the big bang can be probed empirically or not. This raises the stakes and enhances the importance of distinguishing the two scenarios.

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- [1] P. J. Steinhardt and N. Turok, *Science* **296**, 1436 (2002); P. J. Steinhardt and N. Turok, *Phys. Rev. D* **65**, 126003 (2002).
- [2] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981); A. D. Linde, *Phys. Lett. B* **108**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [3] J. Bardeen, P.J. Steinhardt, and M.S. Turner, *Phys. Rev. D* **28**, 679 (1983); V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, *Phys. Reports* **215**, 203 (1992).
- [4] A.H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); S.W. Hawking, *Phys. Lett. B* **115**, 295 (1982); A.A. Starobinskii, *Phys. Lett. B* **117**, 175 (1982).
- [5] D. Spergel *et al.*, *Astrophys. J. Suppl.* **148** (2003) 175.
- [6] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister, S.P. Trivedi JCAP 0310 (2003) 013.
- [7] L. Susskind, hep-th/0302219; S. Kachru, R. Kallosh, A. Linde, S.P. Trivedi, *Phys.Rev. D* **68**, 046005 (2003).
- [8] For a recent review and references, see R. Brandenberger, hep-th/0210186.
- [9] A.G. Riess, *et al.*, *Astrophys. J.* **116**, 1009 (1998); S. Perlmutter, *et al.*, *Bull.Am.Astron.Soc.* **29** (1997) 1351; S. Perlmutter, *et al.*, *Astrophys.J.* **517**, 565-586 (1999).
- [10] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *Phys. Rev. D* **66**, 046005 (2002).
- [11] A. J. Tolley, N. Turok and P. J. Steinhardt, hep-th/0306109, *Physical Review D*, in press (2004).
- [12] J. Khoury, P. J. Steinhardt and N. Turok, *Phys. Rev. Lett.* **92**, 031302 (2004).
- [13] D. Lyth, *Phys. Lett. B* **524**, 1 (2002).
- [14] R. Kallosh, L. Kofman, and A. Linde, *Phys. Rev. D* **64**, 123523 (2001); R. Kallosh, L. Kofman, A. Linde, and A. Tseytlin, *Phys. Rev. D* **64**, 123524 (2001).
- [15] R. Brandenberger and F. Finelli, *JHEP* 0111, 056 (2001). ADD
- [16] R. Durrer and F. Vernizzi, *Phys. Rev. D* **66**, 083503 (2002); C. Cartier, R. Durrer and E. J. Copeland, hep-th/0301198.
- [17] J. Martin, P. Peter, N. Pinto-Neto, D. J. Schwarz, *Phys. Rev. D* **65**, 123513 (2002); **67**, 028301 (2003); J. Martin and P. Peter, astro-ph/0312488.
- [18] G. T. Horowitz and J. Polchinski, *Phys. Rev. D* **66**, 103512 (2002); H. Liu, G. Moore and N. Seiberg, *JHEP* 0206, 045 (2002); D. Lyth, *Phys. Lett. B* **524**, 1 (2002); M. Fabinger and J. McGreevy, hep-th/0206196; M. Fabinger and S. Hellerman, hep-th/0212223.
- [19] J. Khoury, B. A. Ovrut, P. J. Steinhardt, N. Turok, hep-th/0105212; see also <http://feynman.princeton.edu/steinh/cyclicFAQS>.
- [20] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, *Phys. Rev. D* **65**, 086007 (2002). See also M. Berkooz and B. Pioline, hep-th/0307280.
- [21] S. Gratton, J. Khoury, P. J. Steinhardt and N. Turok, astro-ph/0301395.
- [22] L. A. Boyle, P. J. Steinhardt and N. Turok, to appear (2004).
- [23] J. K. Erickson, D. H. Wesley, P. J. Steinhardt and N. Turok, hep-th/0312009.
- [24] J. Dai, R.G. Leigh and J. Polchinski, *Mod. Phys. Lett. A* **4** (1989) 2073; P. Hořava and E. Witten, *Nucl. Phys. B* **460** (1996) 506; **B475** (1996) 94; A. Lukas, B.A. Ovrut and D. Waldram, *Nucl. Phys. B* **532** (1998) 43; *Phys. Rev. D* **57** (1998) 7529; A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Phys. Rev. D* **59** 086001 (1999).
- [25] N. Turok and P.J. Steinhardt, hep-th/0403020.
- [26] P.J. Steinhardt and N. Turok, *Phys. Rev. D* **66**, 101302 (2002).
- [27] L. Susskind, private communication.
- [28] N. Goheer, M. Kleban, and L. Susskind, *JHEP* **0307**, 056 (2003).
- [29] M. Kleban, P.J. Steinhardt, N. Turok, unpublished.
- [30] R. Bouso, *Rev. Mod. Phys.* **74**, 825 (2002).
- [31] S. Gratton, P.J. Steinhardt, N. Turok, unpublished.
- [32] E. M. Lifshitz and I. M. Khalatnikov, *Adv. Phys.* **12**, 185 (1963); V. A. Belinsky, I. M. Khalatnikov and E. M. Lifshitz, *Adv. Phys.* **19**, 525 (1970); V. A. Belinsky and I. M. Khalatnikov, *Sov. Phys. JETP* **36**, 591 (1973); C.W. Misner, *Phys. Rev. Lett.* **22** 1071 (1969).
- [33] J. P. Peebles, *Comments on Astrophysics* **4**, 53 (1972).
- [34] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *Phys. Rev. D* **64**, 123522 (2001); L. A. Boyle, P. J. Steinhardt and N. Turok, hep-th/0307170.
- [35] B. Craps and B.A. Ovrut, hep-th/0308057.
- [36] T. J. Battefield, S. P. Patil, and R. Brandenberger, hep-th/0401010.
- [37] T. Banks and W. Fischler, hep-th/0212113, hep-th/0310288.
- [38] D. Baumann, P.J. Steinhardt and N. Turok, in preparation.