Time, Being and Existence: Lecture Two
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1 Introduction

One criterion of adequacy for a formal ontology, we have said, is that it should provide a logically perspicuous representation of our commonsense understanding of the world, and not just of our scientific understanding. Now a central feature of our commonsense understanding is how we are conceptually oriented in time with respect to the past, the present and the future, and the question arises as to how we can best represent this orientation. It is inappropriate to represent it in terms of a tenseless idiom of moments or intervals of time of a coordinate system, as is commonly done in scientific theories; for that amounts to replacing our commonsense understanding with a scientific view. A more appropriate representation is one that respects the form and content of our commonsense statements about the past, the present and the future. Formally, this can best be done in terms of a logic of tense operators, or in what is now called tense logic.[1]

The most natural formal ontology for tense logic is conceptual realism. That is because what tense operators represent in conceptual realism are certain *cognitive schemata* regarding our orientation in time that are fundamental to both the form and content of our conceptual activity. Thought and communication, in other words, are inextricably temporal phenomena, and it is the cognitive schemata underlying our use of tense operators that

[1] For an excellent book on tense logic and related philosophical issues, see Prior 1967.

structures that phenomena temporally in terms of the past, the present and the future.

Tense logic is important to conceptual realism as a formal ontology for another reason as well; and that is that it helps us understand the distinction between *being* and *existence*. The problem of the distinction between being and existence is one that must arise, and be resolved, in every formal ontology. Put simply, the problem is:

> Can there be things that do not exist?
> Or is being the same as existence?

In tense logic, the distinction between being and existence can be explicated at least partly in terms of the unproblematic distinction between past, present and future objects, i.e., the distinction between things that did exist, do exist, or will exist, or what we call *realia*, as opposed *existentia*, which is restricted to the things that exist at the time we speak or think, i.e., the time we take to be the present of our commonsense framework. The present, in other words, unlike the tenseless medium of our scientific theories, is indexical, and refers at any moment of time to that moment itself.

In what follows we will develop separate logics for both possibilism and actualism, and then we will extend these logics to both possibilist and actualist versions of quantified tense logic, where by possible objects we mean only *realia*, i.e., the things that did, do, or will exist. These logics will serve not only as essential component parts of our larger framework of conceptual realism as a formal ontology, but also as paradigmatic examples of how different parts or aspects of a formal ontology can be developed independently of constructing a comprehensive system all at once. They also illustrate how the model-theoretic methodology of set theory can be used to guide our intuitions in axiomatically developing the formal systems we construct as part of a formal ontology.

It is important to emphasize here that the logical notations and systems described in these logics will be assumed in all subsequent lectures.

2 Possibilism versus Actualism

The two main parts of metaphysics, we have noted, consists of ontology and cosmology, where ontology is the study of being, and cosmology is the study of the physical universe, i.e., the world of natural objects and the space-time manifold in which they exist. If existence is the mode of being of the natural objects of the space-time manifold—i.e., of "actual" objects—then the question arises as to whether or not being is the same as existence, and how this difference, or sameness, is to be represented in formal ontology. We will call the two positions one can take on this issue possibilism and actualism, respectively.

In **possibilism**, there are objects that do not now exist but could exist in the physical universe, and hence *being* is not the same as *existence*. In **actualism** being is the same as existence.

> Possibilism: *There are* objects (i.e., objects that have *being* or) that possibly exist but that do not in fact exist.
> Therefore: Existence $\neq$ Being.
>
> Actualism: Everything that is (has being) exists.
> Therefore: Existence = Being.

In formal ontology, possibilism is developed as a logic of actual and possible objects. Whatever exists in such a logic has being, but it is not necessary that whatever has being exists; that is, there *can* be things that do not exist. Much depends, of course, on what is meant here by 'can'. Does it depend, for example, on the merely possible existence of objects that never in fact exist in the space-time manifold? Or is there a weaker, less committal sense of modality by which we can say that there *can* exist objects that do not now exist? The answer is there are a number of such senses, all explainable in terms of time or the space-time manifold.

We can explain the difference between being and existence, first of all, in terms of the notion of a local time (*Eigenzeit)* of a world-line of space-time. Within the framework of a possibilist tense logic, for example, *being* encompasses past, present, and future objects, while existence encompasses only those objects that presently exist.[2] No doctrine of merely possible existence is needed in such a framework to explain the distinction between existence and being. We can interpret modality, in other words, so that it *can* be true to say that some things do not exist, namely past and future things

[2]For some philosophers, e.g., Arthur Prior, being encompasses only past and present objects. See Prior 1967, chapter viii.

that do not now exist. In fact, there are potentially infinitely many different modal logics that can be interpreted within the framework of tense logic. In this respect, tense logic provides a paradigmatic framework within which possibilism can be given a logically perspicuous representation as a formal ontology.

Tense logic also provides a paradigmatic framework for actualism as well. Instead of possible objects, actualism assumes that there can be vacuous names, i.e., names that denote nothing. Some names, for example, may have denoted something in the past, but now denote nothing because those things no longer exist; and hence the statement that some things do not exist can be true in a semantic, metalinguistic sense, i.e., as a statement about the denotations, or lack of denotations, of singular terms. What is needed, according to actualism, is not that we should distinguish the concept of existence from the concept of being, but only that we should modify the way that the concept of existence (being) is represented in standard first-order predicate logic with identity. On this view, a first-order logic of existence should allow for the possibility that some of our singular terms might fail to denote an existent object, which, according to actualism, is only to say that those singular terms denote nothing, rather than that what they denote are objects (beings) that do not exist. Such a logic for actualism amounts to what today is called a *logic free of existential presuppositions*, or simply *free logic*.

> The logic of actualism = free logic, i.e. logic free of existential presuppositions regarding the denotations of singular terms.

In what follows we shall first formulate a logic of actual and possible objects in which existence and being are assumed to be distinct concepts that are represented by different universal quantifiers $\forall^e$ and $\forall$, respectively, with the existential quantifiers $\exists^e$ and $\exists$ defined in terms of $\forall^e$ and $\forall$ in the usual way. The free logic of actual objects, where existence is not distinguished from being—but also where it is not assumed that all singular terms denote—is then described as a certain subsystem of the logic of actual and possible objects. Of course, it is only from the perspective of possibilism that the logic of actual objects is to be viewed as a proper subsystem of the logic of being, because, according to possibilism, the logic of being includes the logic of actual objects as well. From the perspective of actualism, the logic of actual objects is all there is to the logic of being.

Although both the free logic of actualism and the logic of possibilism have their most natural applications in tense and modal logic, we will first formulate these logics without presupposing any such larger encompassing framework. We will then describe a framework for tense logic where we distinguish an application of the logic of actual and possible objects from an application of the free logic of actual objects *simpliciter*. After that, we will explain how different modal logics can be interpreted in terms of tense logic, and how an application of the logic of actual and possible objects in modal logic can be distinguished from an application of the free logic of actual objects *simpliciter*.

We will also discuss the kinds of qualifications that are required in the statement of the laws involving the interplay of quantifiers, tenses, and modal operators, or what are called *de re* modalities. The tense-logical frameworks for which these laws are stated provide logically perspicuous representations of the differences between actualism and possibilism, including a restricted version of temporal possibilism where determinate being includes only what did or does exist, leaving the future as an indeterminate realm of nonbeing. Tense logic, as these developments will indicate, is a paradigmatic framework in which to formally represent the differences between actualism and possibilism.

3 The Syntax of a First-Order Logic of Actual and Possible Objects

We will initially consider only the first-order logic of actual and possible objects. Later, after we have considered different formal theories of predication, we will extend the logic into a fuller account of being and existence. We turn first to the syntax of the logic.

As *logical constants*, we have the following:

1. The negation sign: $\neg$
2. The (material) conditional sign: $\rightarrow$
3. The conjunction sign: $\wedge$

4. The disjunction sign: $\vee$

5. The biconditional sign: $\leftrightarrow$

6. The identity sign: $=$

7. The possibilist universal and existential quantifiers: $\forall$, $\exists$

8. The actualist universal and existential quantifiers: $\forall^e$, $\exists^e$

When stating axioms, we will assume that $\neg$, $\rightarrow$, $=$, $\forall$, and $\forall^e$ are the only primitive logical constants, and that the others are defined in terms of these in the usual way. We do this only for the convenience of not having to deal with too many axioms when proving metatheorems.

We take *a formal language* L to be a set of objectual constants and predicates of arbitrary (finite) degrees. In this way every formal language will have the same logical grammar and will differ from other formal languages only in the objectual and predicate constants in that language. Objectual constants are the symbolic counterparts of proper names in this sort of logic, and n-place predicate constants are the symbolic counterparts of n-ary relation expressions, with one-place predicate constants the counterparts of monadic predicates. Whether or not predicate constants stand for concepts or properties and intensional relations depends on what formal theory of predication in a larger framework is assumed, the sort of framework that we will turn to later. Also, whether the use of objectual constants is the best way to represent proper names and singular reference is a matter we will turn to later as well. For now, we note only that this is the standard, modern way to represent proper names.

- **Objectual constants:** symbolic counterparts of proper names.

- n**-place predicate constants:** symbolic counterparts of n-place predicate expressions of natural language, for some natural number n.

- **A formal language** L**:** a set of objectual and predicate constants.

The *objectual terms* of a formal language L are the objectual variables and the objectual constants in that language. Atomic formulas of L are the identity formulas of L, i.e., formulas of the form $a = b$, or the result of concatenating an n-place predicate constant of L with n many singular terms of L.

- **The (objectual) terms of** $L =_{df} \{a : a$ is either an objectual constant in L or an objectual variable$\}$.

- **The atomic formulas of** $L =_{df} \{a = b : a, b$ are terms of $L\} \cup \{F(a_1, ..., a_n)$: for some natural number n, F is an n-place predicate constant in L and $a_1, ..., a_n$ are terms of $L\}$

We use two quantifiers—though only one style of objectual variable—one for quantification over possible objects, or possibilia, and the other for quantification over actual objects. The formulas of a language L are those objects that belong to every set K containing the atomic formulas of L and such that $\neg\varphi$, $(\varphi \to \psi)$, $(\forall x\varphi)$, $(\forall^e x\varphi) \in K$ whenever φ, $\psi \in K$ and x is an objectual variable. As indicated, we use Greek letters as variables for expressions of the syntactical metalanguage (set theory).

- χ **is a (first-order) formula of** L if, and only if, for all sets K, if (1) every atomic formulas of L is in K and (2) for all $\varphi, \psi \in K$ and objectual variables x, $\neg\varphi$, $(\varphi \to \psi)$, $(\forall x\varphi)$, and $(\forall^e x\varphi) \in K$, then $\chi \in K$.

Note: The induction principle for formulas follows from this definition.

Induction Principle: If L is a (formal) language, then if:
(1) every atomic formula of $L \in K$, and
(2) for all $\varphi, \psi \in K$ and all objectual variables x, $\neg\varphi$, $(\varphi \to \psi)$, $(\forall x\varphi)$, and $(\forall^e x\varphi) \in K$,
then every formula of $L \in K$.

4 Set-theoretic Semantics

A set-theoretic semantics for the logic of actual and possible objects is only a mathematical tool. It does not explain the difference between being and existence but merely models it mathematically. In this respect it guides our intuitions about how validity and logical consequence are to be determined in the logic.

A model for a formal language L is characterized in terms a universe U of actual objects, a nonempty domain of discourse of possible objects D

containing the universe of actual objects, and an assignment R of extensions drawn from the domain of discourse to the objectual and predicate constants in L. The extension of an objectual constant is what is denoted by that constant, and the extension of an n-place predicate constant F is the set of n-tuples of objects in the domain that F is understood to be true of in the model.

definition: $\mathfrak{A}$ is a model for a formal language L if, and only if, for some U, D, R,
(1) $\mathfrak{A} = \langle U, D, R \rangle$,
(2) D is a nonempty set,
(3) $U \subseteq D$, and
(4) R is a function on L such that for each objectual constant in L, $R(a) \in D$, and for each natural number n and each n-place predicate constant F in L, $R(F) \subseteq D^n$, i.e., $R(F)$ is a set of n-tuples drawn from D.

The assumption that the domain of discourse of possible objects is not empty can be dropped, and later we will have reasons to do so; but, insofar as the domain is restricted to concrete objects—i.e., it does not include any abstract objects—then, as applied to time, the assumption says only that some object exists at some time or other, which seems appropriate; and as applied to possible worlds of concreta, it says only that some object exist in some world or other, which again seems appropriate.

The notions of **satisfaction** and **truth** of a formula of a language L in a model for L are defined in the usual Tarski manner, except that the satisfaction clause for the actual quantifier applies only to the universe of the model in question, whereas the satisfaction clause for the possible quantifier covers the entire domain of discourse, i.e., the set of possibilia of the model. We will not go into those details here. A formula φ is said to be **logically true** if for some language L of which φ is a formula, φ is true in every model suited to L.

definition: φ is logically true if, and only if, for some language L, φ is a formula of L and φ is true in every model $\mathfrak{A}$ suited to L.

5 Axioms and Theorems in Possibilist Logic

We turn now to an axiomatization of the logic of actual and possible objects as our first-order description of possibilism. We note as well that a first-order logic of actualism is properly contained in this version of possibilism.

Where φ, ψ, χ are formulas, x, y are variables, and a, b are (objectual) terms (variables or constants), we take all instances of the following schemas to be axioms of the logic of actual and possible objects.

(A1) $\varphi \rightarrow (\psi \rightarrow \varphi)$

(A2) $[\varphi \rightarrow (\psi \rightarrow \chi)] \rightarrow [(\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi)]$

(A3) $(\neg\varphi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \varphi)$

(A4) $(\forall x)[\varphi \rightarrow \psi] \rightarrow [(\forall x)\varphi \rightarrow (\forall x)\psi]$

(A5) $(\forall^e x)[\varphi \rightarrow \psi] \rightarrow [(\forall^e x)\varphi \rightarrow (\forall^e x)\psi]$

(A6) $\varphi \rightarrow (\forall x)\varphi$, where x is not free in φ

(A7) $(\forall x)\varphi \rightarrow (\forall^e x)\varphi$

(A8) $(\exists x)(a = x)$, where x is not a

(A9) $(\forall^e x)(\exists^e y)(x = y)$ where x, y are distinct variables

(A10) $a = b \rightarrow (\varphi \rightarrow \psi)$, where φ, ψ are atomic formulas and ψ is obtained from φ by replacing an occurrence of b by a

As inference rules we assume only *modus ponens* and the rule of universal generalization The turnstile, $\vdash$, is read as 'is a theorem of the logic of actual and possible objects'.

MP: If $\vdash \varphi \rightarrow \psi$ and $\vdash \varphi$, then $\vdash \psi$.

UG: If $\vdash \varphi$, then $\vdash (\forall x)\varphi$.

By axiom (A7) and (UG), we also have as a derived rule:

UG^e: If $\vdash \varphi$, then $\vdash (\forall^e x)\varphi$.

Axioms (1)–(A3), as is well-known, suffice to validate all tautologous formulas, which we will assume hereafter. Note that where a, b, c are terms (variables or constants), the transitivity of identity law,

$$\vdash a = b \rightarrow (b = c \rightarrow a = c),$$

is an immediate consequence of axiom (A10). The reflexive law of self-identity,

$$\vdash a = a,$$

is a consequence of axioms (A10), (UG), (A4), (A6) (A8) and tautologous transformations.[3] The symmetry of identity law for identity then follows from (A10), transitivity and reflexivity.[4] Finally, note that by a simple induction on formulas, Leibniz's law, (LL),

$$\vdash a = b \rightarrow (\varphi \leftrightarrow \psi), \qquad (\mathbf{LL})$$

where ψ is obtained from φ by replacing one or more *free occurrences* of a by free occurrences of b, is provable.[5] In other words, the full logic of identity, which we will assume hereafter, is contained in this system.

Note that the principle of universal instantiation,

$$(\forall x)\varphi \rightarrow \varphi(a/x), \qquad (\mathbf{UI})$$

where a can be properly substituted for x in φ, is not one of our axiom schemas.[6] This is convenient because when extending the logic by adding tense and modal operators we generally have to revise this principle when taken as an axiom so as to cover the cases when x has *de re* occurrences in φ, i.e., when x occurs within the scope of a tense or modal operator—or any other intensional operator. The restrictions needed in each case can be

[3]That is, where a and y are distinct terms, then by (A10), $\vdash a = y \rightarrow (a = y \rightarrow a = a)$, and hence, $\vdash a = y \rightarrow a = a$, and therefore $\vdash a \neq a \rightarrow a \neq y$. Thus, by (UG) and (A4), $\vdash (\forall y)(a \neq a) \rightarrow (\forall y)(a \neq y)$,. But by (A6), $\vdash a \neq a \rightarrow (\forall y)(a \neq a)$, and hence $\vdash a \neq a \rightarrow (\forall y)(a \neq y)$. That is, by tautology, $\vdash \neg(\forall y)(a \neq y) \rightarrow a = a$. But, by (A8) and the definition of $\exists$, $\vdash \neg(\forall y)(a \neq y)$, from which it follows that $\vdash a = a$.

[4]THat is, by (A10), $\vdash a = b \rightarrow (a = a \rightarrow b = a)$. But $\vdash a = a$, so therefore by tautology and modus ponens, $\vdash a = b \rightarrow b = a$.

[5]The case for atomic formulas is a consequence of (A10), the symmetry of identity law, and tautologous transformations. The other cases follow by the inductive hypothesis, tautologous transformations, and finally (A4) and (A5).

[6]The notion of proper substitution of a term for a variable that is needed for **(UI)** is also not involved in any of the axioms; and even the notion of bondage and freedom in (A6) can be replaced by the notion of an occurrence *simpliciter*. This is very convenient because these notions are complex and difficult for students to grasp at first. Also, it is convenient to avoid these notions when using Gödel's arithmetization technique, because they add so much complexity to that technique. A yet further, and perhaps even more important reason is noted above.

determined by seeing what is needed in the proof by induction of Leibniz's law, **(LL)**, from which this principle follows. That is, by **(LL)**,

$$\vdash a = x \rightarrow [\varphi \rightarrow \varphi(a/x)],$$

where $\varphi(a/x)$ is the result of replacing *all* free occurrences of x by a (which we assume to be distinct from x). Then, by tautologous transformations, **(UG)**, (A4) and (A6), we have

$$\vdash (\exists x)(a = x) \rightarrow [(\forall x)\varphi \rightarrow \varphi(a/x)],$$

from which, by axiom (A8) and modus ponens, **(UI)** follows. The law for existential generalization,

$$\varphi(a/x) \rightarrow (\exists x)\varphi, \qquad \textbf{(EG)}$$

is of course the converse of **(UI)** and therefore provable on its basis.

Axiom (A8) is the important axiom here. One should not think that it is redundant because it is provable by **(EG)** from the law of self-identity. Of course,

$$a = a \rightarrow (\exists x)(a = x)$$

is an an instance of **(EG)**; but **(EG)** is provable from **(UI)**, and, as noted, in the proof of **(UI)** we need (A8), i.e., $(\exists x)(a = x)$. So it would be circular reasoning to try to prove the latter in terms of **(EG)**.

What axiom (A8) says in effect is that every objectual term denotes a possible object (as a value of the bound objectual variables), even if that object does not *exist* (as a value of the variables bound by the actualist quantifier, $\forall^e$). That is not a problem if the logic does not introduce complex objectual terms, i.e., objectual terms that might contain formulas that describe impossible situations (such as 'the round square'), or if the logic is not extended to a situation where certain complex objectual terms must fail to denote on pain otherwise of resulting in a contradiction. (This is what in fact happens, in the next lecture, when we extend the logic to second-order predicate logic with nominalized predicates as abstract objectual terms.) It is this latter situation that we will later be concerned with, in which case we will then have to replace axiom (A8) by $(\forall x)(\exists y)(x = y)$, which is the possibilist counterpart of the actualist axiom (A9).

Finally, we note that If we restrict ourselves to formulas in which the actualist quantifier does not occur, then axioms (A1)-(A4), (A6), (A8), and

(A10) yield all and only the standard logical truths of first-order predicate logic.[7] The standard logical truths, moreover, are none other than the logical truths as defined above when formulas are restricted to those in which the actualist quantifier does not occur. What these results show, in other words, is that the logic of possible objects is none other than standard first-order predicate logic with identity. Hereafter we will assume all the known results about this logic.

The main result of this section is that a formula is a theorem of the logic of actual and possible objects if, and only if, it is logically true.

Metatheorem: For all formulas φ, $\vdash \varphi$ if, and only if, φ is logically true.

6 A First-order Actualist Logic

Essentially the same proof that we described above for **(UI)** applies to the principle of universal instantiation for the actualist quantifier, except that now the antecedent condition in one step of that proof is not an axiom. That is, by Leibniz's law, **(UG)**, (A5), the counterpart of (A6) for $\forall^e$ (which is derivable from (A6) and (A7)), we have

$$(\exists^e y)(a = y) \rightarrow [(\forall^e x)\varphi \rightarrow \varphi(a/x)], \qquad (\mathbf{UI}^e)$$

which is the actualist version of universal instantiation (where a and y are distinct terms). We cannot assume the antecedent here unless we are given as a separate premise that the objectual term a denotes an actual, existent, object (as a value of the variable bound by $\exists^e$). Note that in addition to the quantifier concept of existence represented by $\forall^e$ (and its dual $\exists^e$), the predicable concept of existence can be defined in this first-order logic as follows (where a is distinct from the variable y):

$$E!(a) =_{df} (\exists^e y)(a = x).$$

Now by an E-formula, let us understand a formula in which the possible quantifier does not occur.

[7] The completeness of the system as described above is due to D. Kalish and R. Montague 1965, their result being obtained by a modification of an original formulation by A. Tarski.

Definition: φ **is an** E**-formula** $=_{df}$ φ is a formula in which the possibilist quantifier, $\forall$, does not occur.

Note that if we restrict ourselves to E-formulas, then neither

(A6^e) $\varphi \rightarrow (\forall^e x)\varphi$, where x is not free in φ,

nor

(A8^e) $a = a$, where a is a objectual term

are provable. That is, the proofs of these valid formulas depend on several possibilist axioms, namely, (A6), (A7) and (A8). That means that if we want to consider only actualism, and not the full logic of possible and actual objects, then we need to take both of these schemas as axioms of the logic of actualism. Indeed, it can be shown that axioms (Al)-(A3), (A5), (A9), (A10), together with these schemas, (A6^e) and (A8^e), yield all and only those logical truths that are E-formulas.[8]

Metatheorem: All logical truths that are E-formulas are derivable from axiom schemas (Al)-(A3), (A5), (A9), (A10), (A6^e) and (A8^e) with modus ponens and **(UG^e)** as the only inference rules.

Thus, whereas

1. the standard formulas that are logically true constitute *the logic of possible objects* simpliciter,
2. the E-formulas that are logically true constitute *the logic of actual objects* simpliciter.
3. All of the formulas together—i.e., the standard formulas, the E-formulas, and the formulas that contain both the possible and the actual quantifiers—that are logically true constitute *the logic of actual and possible objects*, which can be shown to be complete for the semantics described above.

[8] See Cocchiarella 1966.

7 Tense Logic

As already indicated, one of the most natural applications of the logic of actual and possible objects is in tense logic, where existence applies only to the things that presently exist, and possible objects are none other than past, present, or future objects, i.e., objects that either did exist, do exist, or will exist. The most natural formal ontology for tense logic is conceptual realism. This is because as forms of conceptual activity, thought and communication are inextricably temporal phenomena, and to ignore this fact in the construction of a formal ontology is to court possible confusion of the Platonic with the conceptual view of intensionality.

Propositions on the conceptualist view, for example, are not abstract entities existing in a platonic realm independently of all conceptual activity. Rather, according to conceptual realism, they are conceptual constructs corresponding to a projection on the level of objects of the truth-conditions of our temporally located assertions. However, on our present level of analysis, where propositional attitudes are not being considered, their status as constructs can be left completely in the metalanguage.

What is also a construction, but which, should not be left to the metalanguage, are certain cognitive schemata characterizing our conceptual orientation in time and implicit in the form and content of our assertions as mental acts. These schemata, whether explicitly recognized as such or not, are usually represented or modelled in terms of a tenseless idiom (such as our set-theoretic metalanguage) in which reference can be made to moments or intervals of time (as objects of a special type). Of course, for most scientific purposes such a representation is quite in order. But to represent them only in this way in a context where our concern is with a perspicuous representation of the form of our assertions as mental acts might well mislead us into thinking that the schemata in question are not essential to the form and content of an assertion after all—the way they are not essential to the form and content of a proposition on the Platonic view.

Now it is important to note that even though the cognitive schemata in question can be modelled in terms of a tenseless idiom of moments or intervals of time, as in fact they are in our set-theoretic metalanguage, they are really themselves the conceptually prior conditions that lead to the construction of our referential concepts for moments or intervals of time. In other words, in terms of conceptual priority, these cognitive schemata are implicitly presupposed by the same tenseless idiom in which they are set-theoretically modelled. In this regard, despite the use of moments or intervals of time in the semantic clauses of the metalanguage, there is no need for conceptualism to assume that moments or intervals of time are independently existing objects as opposed to being merely constructions out of the different events that actually occur in nature, constructions that can be given within a tense logically based language.

Now because the temporal schemata implicit in our assertions enable us to orientate ourselves in time in terms of the distinction between the past, the present, and the future, a more appropriate or perspicuous representation of these schemata is one based upon a system of quantified tense logic where we represent tenses by means of tense operators. As applied in thought and communication, what these operators correspond to is our ability to refer to what was the case, what is the case, and what will be the case—and to do so, moreover, without having first to construct referential concepts for moments or intervals of time. In the simplest case we have operators only for the past and the future.

$\mathcal{P}$	it **was** the case that ...
$\mathcal{F}$	it **will be** the case that ...

We do not need an operator for the simple present tense, 'it is the case that', because it is already represented in the simple indicative mood of our predicates. Note that with negation applied both before and after a tense operator, we can shorten the long reading of $\neg\mathcal{P}\neg$, namely, 'it was not the case that it was the case that it was not the case' to simply 'it was always the case'. A similar shorter reading applies to $\neg\mathcal{F}\neg$ as well. In other words,. we also have the following readings:

$\neg\mathcal{P}\neg$	it **always was** the case that ...
$\neg\mathcal{F}\neg$	it **always will be** the case that ...

Tensed formulas are defined inductively as follows.

Definition: φ **is a tensed formula of a language** L if, and only if, φ is in every set K such that (1) every atomic formula of L is in K, and (2) whenever $\varphi, \psi \in K$ and x is a variable, then $\neg\varphi$, $(\varphi \rightarrow \psi)$, $\mathcal{P}\varphi$, $\mathcal{F}\varphi$, $(\forall x)\varphi$, $(\forall^e x)\varphi \in K$.

We will avoid going into all of the details here of a set-theoretical semantics for tense logic.[9] Briefly, the idea is that we consider the earlier-than

[9]For such details see Cochiarella 1966 or 1974.

relation of a local time (*Eigenzeit*) of a world-line in space-time. Though it is natural to assume that this relation is a serial ordering, i.e., that it is transitive, asymmetric and connected, we initially impose no constraints at all upon it, and then consider validity with respect to the different kinds of structures that it can have, e.g., that it is discrete, dense, or continuous, has a beginning and end, or neither, etc. What is important is that we distinguish the objects that exist at each point of the world line from the objects that exist at any of the points of the line. Formally, this can stated as follows.

Definition: If L is a (formal) language and $\mathfrak{A}$ is a model (as defined earlier) for a language L, then
(1) U_{A} = the universe (of existing objects) of $\mathfrak{A}$, and
(2) D_{A} = the set of possibilia of $\mathfrak{A}$.

Definition: If L is a language and R is a serial relation, then B **is an** R**-history with respect to** L if there are a nonempty index set I included in the field of R and an I-termed sequence $\mathfrak{A}$ of models suited to L such that
(i) $\mathfrak{B} = \langle R, \mathfrak{A}_{i \in I} \rangle$;
(ii) I is identical with the field of R if I has more than one element; and
(iii) $\cup_{j \in I} U_{\mathsf{A}_j} \subseteq D_{\mathsf{A}_i}$, for all $i \in I$; and
(iv) $D_{\mathsf{A}_i} = D_{\mathsf{A}_j}$, for all $i, j \in I$.

Where $\langle R, \mathfrak{A}_{i \in I} \rangle$ is such a history, we take the members of the set I to be the **moments** of the history and R to be **the earlier-than relation** ordering those moments. The structure of R is the temporal structure of the history. It may, for example, have a beginning, or an end, both, or neither, and it may be discrete, dense, or continuous, and so on. Condition (iii) stipulates that whatever is actual at one time or another in a history is a possible object of that history. Condition (iv) states the requirement that whatever is a possible object at one moment of a history is a possible object at any other moment of that history. A complete description of the world relative to the language L and a moment i of a history $\langle R, \mathfrak{A}_{i \in I} \rangle$ is given by the model $\mathfrak{A}_i$ that is associated with i in that history.

Except where tense operators are involved, *satisfaction* and *truth* in a history $\langle R, \mathfrak{A}_{i \in I} \rangle$ at a given moment i of the history is understood as satisfaction and truth in the model $\mathfrak{A}_i$. The satisfaction clauses for the tense operators have the obvious references to the models associated with the moments before and after the moment i. *Validity* in a history is defined as truth at all times in that history. If R is a relation, then φ is said to be R-*valid* if φ is a tensed formula of some language L such that for each R-history $\mathfrak{B}$ with respect to L, φ is valid in $\mathfrak{B}$. A tensed schematic formula φ is understood to characterize a class K of relations if for each relation R, φ is R-valid if, and only if, $R \in K$.[10]

Definition: If φ is a tensed formula of a language L and $\mathfrak{B} = \langle R, \mathfrak{A}_{i \in I} \rangle$ is an history, then:
(1) φ **is valid in** $\mathfrak{B}$ if, and only if, for all $i \in I$, φ is true at i in $\mathfrak{B}$;
(2) φ **is** R**-valid** if, and only if, for each R-history $\mathfrak{B}'$, φ is *valid in* $\mathfrak{B}'$; and
(3) φ **characterizes a class** K **of relations** if, and only if, for each relation R, φ is R-*valid* if, and only if, $R \in K$.

Special schematic formulas can be shown to characterize various classes of relations. For example, the tensed formulas

$$\mathcal{PP}\varphi \to \mathcal{P}\varphi$$

$$\mathcal{FF}\varphi \to \mathcal{F}\varphi$$

characterize the class of transitive relations, whereas the converse schemas

$$\mathcal{P}\varphi \to \mathcal{PP}\varphi$$

$$\mathcal{F}\varphi \to \mathcal{FF}\varphi$$

characterize the class of dense relations, i.e., where between any two moments of time there is always another moment of time.

The most natural assumption, we have said, is that the earlier-than relation of a local time (*Eigenzeit*) of a world-line is a serial ordering, i.e. that the relation is transitive, asymmetric and connected. The claim that Einstein's theory of special relativity shows that connectedness does not apply is based on a confusion of

1. *the causal signal relation*, which is not connected, between the different momentary states of different world lines with

[10] See Cocchiarella 1966 and 1974 for the details of this semantics.

2. *the earlier-than relation* of a local time *(Eigenzeit)* of a given world line, which is connected.

In any case, in regard to the characterization of logical truth as extended to all tensed formulas, we restrict our considerations—in deference to this fundamental feature of (local) time—to serial histories, i.e., histories whose temporal ordering is a series.

Definition: φ **is tense-logically true** if for some language L of which φ is a tensed formula, φ is valid in every serial history suited to L.

Metatheorem: φ is tense-logically true if, and only if, for every *serial ordering* R, φ is R-valid.

Given modus ponens and universal generalization for $\forall$, and the following as inference rules

(i) if $\vdash_t \varphi$, then $\vdash_t \neg\mathcal{P}\neg\varphi$

(ii) if $\vdash_t \varphi$, then $\vdash_t \neg\mathcal{F}\neg\varphi$

then these rules together with all instances of (A1)-(A10) of the logic of actual and possible objects (applied now to tensed formulas) and all instances of the following axiom schemas as well yield all and only the tense-logical truths:

(A11) $\neg\mathcal{P}\neg(\varphi \to \psi) \to (\mathcal{P}\varphi \to \mathcal{P}\psi)$

(A12) $\neg\mathcal{F}\neg(\varphi \to \psi) \to (\mathcal{F}\varphi \to \mathcal{F}\psi)$

(A13) $\varphi \to \neg\mathcal{P}\neg\mathcal{F}\varphi$

(A14) $\varphi \to \neg\mathcal{F}\neg\mathcal{P}\varphi$

(A15) $\mathcal{P}\mathcal{P}\varphi \to \mathcal{P}\varphi$

(A16) $\mathcal{F}\mathcal{F}\varphi \to \mathcal{F}\varphi$

(A17) $\mathcal{P}\varphi \wedge \mathcal{P}\psi \to \mathcal{P}(\varphi \wedge \psi) \vee \mathcal{P}(\varphi \wedge \mathcal{P}\psi) \vee \mathcal{P}(\psi \wedge \mathcal{P}\varphi)$

(A18) $\mathcal{F}\varphi \wedge \mathcal{F}\psi \to \mathcal{F}(\varphi \wedge \psi) \vee \mathcal{F}(\varphi \wedge \mathcal{F}\psi) \vee \mathcal{F}(\psi \wedge \mathcal{F}\varphi)$

(A19) $x = y \to \neg\mathcal{P}\neg(x{=}y) \wedge \neg\mathcal{F}\neg(x{=}y)$ where x, y are variables

Metatheorem: For all tensed formulas φ, $\vdash_t \varphi$ if, and only if, φ is tense-logically true.[11]

[11] See Cocchiarella 1966 for a proof of this metatheorem.

A completeness theorem for actualist tensed logic is also forthcoming if we restrict ourselves to tensed E-formulas, i.e., those formulas in which the possible quantifier does not occur. Restricted to tensed E-formulas, the axioms for actualism are as follows:

Tensed Actualist axioms: (A1)-(A3), (A5), (A6^e), (A8^e), (A9), and (A10)-(A19).

We will also need the inference rules listed above—but with universal generalization for $\forall^e$ instead of $\forall$—as well as the following (somewhat complex) rules:

(iii) If $\vdash_t \neg\mathcal{P}\neg(\varphi_1 \rightarrow \neg\mathcal{P}\neg(\varphi_1 \rightarrow ... \rightarrow \neg\mathcal{P}\neg(\varphi_{n-1} \rightarrow \neg\mathcal{P}\neg\varphi_n)...))$,
and x is not free in $\varphi_1, ..., \varphi_{n-1}$, then
$\vdash_t \neg\mathcal{P}\neg(\varphi_1 \rightarrow \neg\mathcal{P}\neg(\varphi_1 \rightarrow ... \rightarrow \neg\mathcal{P}\neg(\varphi_{n-1} \rightarrow \neg\mathcal{P}\neg(\forall^e x)\varphi_n)...))$.

(iv) If $\vdash_t \neg\mathcal{F}\neg(\varphi_1 \rightarrow \neg\mathcal{F}\neg(\varphi_1 \rightarrow ... \rightarrow \neg\mathcal{F}\neg(\varphi_{n-1} \rightarrow \neg\mathcal{F}\neg\varphi_n)...))$,
and x is not free in $\varphi_1, ..., \varphi_{n-1}$, then
$\vdash_t \neg\mathcal{F}\neg(\varphi_1 \rightarrow \neg\mathcal{F}\neg(\varphi_1 \rightarrow ... \rightarrow \neg\mathcal{F}\neg(\varphi_{n-1} \rightarrow \neg\mathcal{F}\neg(\forall^e x)\varphi_n)...))$.

The above axioms and rules yield all and only those tense-logical truths that are tensed E-formulas.[12]

8 Temporal Modes of Being

In assuming that being and existence are not the same concept, possibilism does not also assume that whatever is (i.e., whatever has being) either did exist, does exist, or will exist, a thesis we shall call *temporal possibilism*. We will call the objects of temporal possibilism **realia**. Formally, this thesis is stated as follows:

Temporal Possibilism: $(\forall x)[\mathcal{P}E!(x) \vee E!(x) \vee \mathcal{F}E!(x)]$.

Realia: What did, does, or will exist.

If we add this formula as a new axiom, then to render it tense-logically true we need only require that the condition stated in clause (iii) of the definition of an R-history be an identity rather than just an inclusion.

[12] See Cocchiarella 1966.

It should be noted that Aristotle seems to have held such a view in that he thought that whatever is possible is realizable in time, which, for Aristotle, has no beginning or end.

A more restrictive view than temporal possibilism—but one that still falls short of actualism—is that only the past and the present are metaphysically determinate, and for that reason only objects that either do exist or did exist have being. The future, being indeterminate metaphysically as well as epistemically has no being. Being, on this account, covers only past or present existence. Future objects have no being but only come into being in the present when they exist, and then continue to have being in the past. We can characterize this position by first defining quantification over past objects, and then quantification over past and present objects, as follows (where $\exists^p$ and $\exists^p_p$ are defined in the usual way as the duals of $\forall^p$ and $\forall^p_p$, respectively):

$$(\forall^p x)\varphi =_{df} (\forall x)[\mathcal{P}E!(x) \rightarrow \varphi]$$

$$(\forall^p_p x)\varphi =_{df} (\forall x)[\mathcal{P}E!(x) \vee E!(x) \rightarrow \varphi]$$

The metaphysical thesis that being comprises only what either did exist or does exist can now be expressed as follows:

$$(\forall x)(\exists^p_p y)(x = y).$$

Alternatively, instead of having the concept of being in such a framework represented by the possibilist quantifier $\forall$, we can take it to be represented directly by $\forall^p_p$ as a primitive quantifier together with $\forall^e$ for the concept of existence.

A sound and complete axiom set for this system is then given by (A1)-(A3), (A5), (A9), (A10), together with the schemas:

$(\forall^p_p x)(\varphi \rightarrow \psi) \rightarrow [(\forall^p_p x)\varphi \rightarrow (\forall^p_p x)\psi]$,

$\varphi \rightarrow (\forall^p_p x)\varphi$, where x is not free in φ,

$(\forall^p_p x)\varphi \rightarrow (\forall^e x)\varphi$,

$(\forall^p_p x)(\exists^p_p y)(x = y)$, where x, y are distinct variables,

$(\forall^p_p x)[(\exists^p_p y)(x = y) \wedge \neg\mathcal{F}\neg(\exists^p_p y)(x = y)]$,

$(\forall^p_p x)\neg\mathcal{P}\neg\varphi \rightarrow \neg\mathcal{P}\neg(\forall^p_p x)\varphi$,

$\neg\mathcal{F}\neg(\forall^p_p x)\varphi \rightarrow (\forall^p_p x)\neg\mathcal{F}\neg\varphi$,

$a = a,$ where a is an arbitrary term.

The inference rules for this system are modus ponens, universal generalization for $\forall^p_p$, rules (i), (ii) as described earlier, and the counterpart of rule (iv) using $\forall^p_p$ in place of $\forall^e$.[13]

9 Referring to Past and Future Objects

Actualists claim that quantificational reference to either past or future objects is possible only *indirectly* (*de dicto*)—i.e., through the occurrence of an actualist quantifier within the scope of a tense operator.[14] This is true of some of our references to past objects, as for example in an assertion of,

Someone did exist who was a King of France.

In this case, the apparent reference to a past object can be accounted for as follows:

$$\mathcal{P}(\exists^e x)King\text{-}of\text{-}France(x),$$

where the reference to a past object is not direct but indirect, i.e., within the scope of a past-tense operator. Here, by *a direct quantificational reference* to a past object that no longer exists (or a future one that has yet to exist) we mean one in which the quantifier is outside the scope of a tense operator.

Note: What is apparently not possible, on this account, about a *direct* quantificational reference to past objects that no longer exist is our present inability to actually confront and apply the relevant identity criteria to objects that do not now exist.

A present ability to identify past or future objects, however, is not the same as the ability to actually confront and identify those objects in the present; that is, our existential inability to do the latter is not the same as, and should not be confused with, what is only presumed to be our inability to directly refer to past or future objects. Indeed, the fact is that we can and do make *direct reference* to *realia*, and to past and future objects in

[13] The counterpart of rule (iii) with $\forall^p_p$ in place of $\forall^e$ is provable, and it does not need to be taken as a primitive rule in this system.

[14] Cf. Prior, 1967, Chapter 8.

particular, and that we do so not only in ordinary discourse but also, and especially, in most if not all of our scientific theories.

The real problem is not that we cannot directly refer to past and future objects, but rather how it is that conceptually we come to do so.

One explanation of how this comes to be can be seen in the analysis of the following English sentences:

1. There did exist someone who is an ancestor of everyone now existing.
2. There will exist someone who will have everyone now existing as an ancestor.

Assuming, for simplicity, that we are quantifying only over persons, it is clear that (1) and (2) cannot be represented by:

3. $\mathcal{P}(\exists^e x)(\forall y) Ancestor\text{-}of(x, y)$
4. $\mathcal{F}(\exists^e x)(\forall y) Ancestor - of(y, x)$.

What (3)and (4) represent are the different sentences:

5. There did exist someone who was an ancestor of everyone *then* existing.
6. There will exist someone who will have everyone *then* existing as an ancestor.

Of course, in temporal possibilism, referential concepts are available that enable us to refer directly to past and future objects. Thus, for quantification over past objects we have the quantifier $\exists^p$ and for quantification over future objects we have $\exists^f$ as the future-counterpart of $\exists^p$. Using these quantifiers, the obvious representation of (1) and (2) is:

7. $(\exists^p x)(\forall y) Ancestor\text{-}of(x, y)$
8. $(\exists^f x)(\forall y)\mathcal{F} Ancestor\text{-}of(y, x)$.

We should note here that the relational ancestor concept is such that:

> x *is* an ancestor of y only at those times when either y exists and x did exist, though x need not still exist at the time in question, or when x has continued to exist even though y has ceased to exist.
>
> When y no longer exists as well as x, we say that x *was* an ancestor of y; and where y has yet to exist, we say that x *will be* an ancestor of y.

Now although these last analyses are not available in actualist tense logic, nevertheless semantical equivalences for them are available once we allow us the use of *the now-operator*, $\mathcal{N}$.

$\mathcal{N}$: It is now the case that ...

The now-operator is unlike the simple present tense in that it always brings us back to the present even when it occurs within the scope of either the past- or future-tense operators. Thus, although the indirect references to past and future objects in (3) and (4) fail to provide adequate representations of (1) and (2), the same *indirect* references followed by the now-operator succeed in capturing the direct references given in (7) and (8):

9. $\mathcal{P}(\exists x)\mathcal{N}(\forall y)Ancestor\text{-}of(x, y)$
10. $\mathcal{F}(\exists x)\mathcal{N}(\forall y)\mathcal{F}Ancestor\text{-}of(y, x)$.

In other words, at least relative to any present-tense context, we can in general account for direct reference to past and future objects, and hence to all of the objects of temporal possibilism, as follows:

$$(\forall^p x)\varphi \leftrightarrow \neg\mathcal{P}\neg(\forall^e x)\mathcal{N}\varphi$$

$$(\forall^f x)\varphi \leftrightarrow \neg\mathcal{F}\neg(\forall^e x)\mathcal{N}\varphi$$

$$(\forall x)\varphi \leftrightarrow (\forall^p x)\varphi \wedge \varphi \wedge (\forall^f x)\varphi.$$

These equivalences, it should be noted, cannot be used other than in a present tense context; that is, the above use of the now-operator would be inappropriate when the equivalences are stated within the scope of a past- or future-tense operator, because in that case the direct reference to past or future objects would be from a point of time other than the present. Formally, what is needed in such a case is the introduction of a so-called "backwards-looking" operator, such as *the then-operator*, which can be correlated with occurrences of past or future tense operators within whose scope they lie and that semantically evaluate the formulas to which they are themselves applied in terms of the past or future times already referred to by the tense operators they are correlated with[15]. Backwards-looking operators, in other words, enable us to conceptually return to a past or future time already referred to in a given context in the same way that the now-operator enables us to return to the present. In that regard, their role in the cognitive schemata characterizing our conceptual orientation in time and implicit in each of our assertions is essentially a projection of the role of the now-operator.

We will not formulate the semantics of these backwards-looking operators here. But we do want to note that by means of such operators we can account for the development of referential concepts by which we can refer directly to past or future objects. Such an account is already implicit in the fact that such direct references to past or future objects can be made with respect to the present alone. This shows that whereas the reference is direct at least in effect, nevertheless the application of any identity criteria associated with such reference will itself be indirect, and in particular, not such as to require a present confrontation, even if only in principle, with a past or future object.

10 Modality Within Tense Logic

It is significant that the first modal concepts to be discussed and analyzed in the history of philosophy are concepts based on the distinction between the past, the present, and the future, that is, concepts that can be analyzed in terms of the temporal-modalities that are represented by the standard tense operators. The Megaric logician Diodorus, for example, is reported as having argued that the possible is that which either is or will be the case, and that the necessary is that which is and always will be the case.[16] Formally, the Diodorean modalities can be defined as follows:

$$\Diamond^f \varphi =_{df} (\varphi \vee \mathcal{F}\varphi)$$

$$\Box^f \varphi =_{df} \varphi \wedge \neg\mathcal{F}\neg\varphi$$

$$\therefore \Box^f \varphi \leftrightarrow \neg\Diamond^f \neg\varphi$$

[15] Cf. Vlach,1973 and Saarinen 1976.

[16] See Prior 1967, chapter 2, for a discussion of Diodorus's argument.

Aristotle, on the other hand, included the past as part of what is possible; that is, for Aristotle the possible is that which either was, is, or will be the case (in what he assumed to be the infinity of time), and therefore the necessary is what is always the case[17]:

$$\Diamond^t\varphi =_{df} \mathcal{P}\varphi \vee \varphi \vee \mathcal{F}\varphi$$

$$\Box^t\varphi =_{df} \neg\mathcal{P}\neg\varphi \wedge \varphi \wedge \neg\mathcal{F}\neg\varphi$$

$$\therefore \Box^t\varphi \leftrightarrow \neg\Diamond^t\neg\varphi$$

Both Aristotle and Diodorus assumed that time is real and not ideal. In other words, the Diodorean and Aristotelian temporal modalities are understood to be *real modalities* based on the nature of time. In fact they provide a paradigm by which we might understand what is meant by a real, as opposed to a merely formal, modality such as logical necessity. These temporally-based modalities contain an explanatory, concrete interpretation of what is sometimes called the accessibility relation between possible worlds in modal logic, except that worlds are now construed as momentary states of the universe as described by the models associated with the moments of a local time. That is, where possible worlds are momentary descriptive states (models) of the universe with respect to the local time (*Eigenzeit*) of a given world-line, then the relation of accessibility between worlds is ontologically grounded in terms of the earlier-than relation of that local time.

The Aristotelian modalities are stronger than the Diodorean, of course, and in fact they provide a complete semantics for the quantified modal logic known as $S5$.

Definition: If L is a language, then φ is an **S5^t-formula** of L if, and only if, φ belongs to every set K containing the atomic formulas of L and such that $\neg\varphi$, $\Box^t\varphi$, $\Diamond^t\varphi$, $(\varphi \to \psi)$, $(\forall x)\varphi$, $(\forall^e x)\varphi \in K$ whenever $\varphi, \psi \in K$ and x is a variable.

Definition: φ is **S5-valid** if, and only if, φ is an **S5**^{t}-formula that is tense-logically true.

[17] See Hintikka, 1973, Chapters V and IX. Aristotle may have intended his notion of possible to apply to individuals as well, a position that is validated in the quantified modal logic described in this section.

We obtain the system we call $\mathbf{S5}^t$ if to the axioms (A1)-(A10) of the logic of actual and possible objects we add all instances of schemas of the following forms:

($\mathbf{S5}^t$-1) $\quad \Box^t\varphi \to \varphi$
($\mathbf{S5}^t$-2) $\quad \Box^t(\varphi \to \psi) \to (\Box^t\varphi \to \Box^t\psi)$
($\mathbf{S5}^t$-3) $\quad \Diamond^t\varphi \to \Box^t\Diamond^t\varphi$
($\mathbf{S5}^t$-4) $\quad (x = y) \to \Box^t(x = y)$, where x, y are variables.

As inference rules we take in addition to modus ponens and universal generalization for $\forall$ the following:

(MG) if $\vdash_{\mathbf{S5}^t} \varphi$, then $\vdash_{\mathbf{S5}^t} \Box^t\varphi$.

Note that by ($\mathbf{S5}^t$-4), (A8) and **(MG)**,

$$(\exists y)\Box^t(x = y)$$

is provable, and from this it can be shown that the Carnap-Barcan formula,

$$(\forall x)\Box^t\varphi \leftrightarrow \Box^t(\forall x)\varphi$$

is also provable. A completeness theorem for this version of quantified **S5** modal logic can be proved in the usual way.

Metatheorem: For each $\mathbf{S5}^t$-formula φ, $\vdash_{\mathbf{S5}^t} \varphi$ if, and only if, φ is **S5**-valid.

For an actualist **S5** modal logic, we need only restrict the **S5**-formulas to those that are tensed E-formulas—i.e., tensed formulas in which the possibilist quantifier does not occur—and use only the logic of actual objects as described earlier together with the axiom schemas ($\mathbf{S5}^t$-1)-($\mathbf{S5}^t$-4) and one new inference rule added to those of $\mathbf{S5}^t$. That is, where $\mathbf{S5}^t_e$ is that subsystem of $\mathbf{S5}^t$ that is the result of replacing (Al)-(A10) of the logic of actual and possible objects by the axioms for the logic of actual objects *simpliciter* and adding to the inference rules of $\mathbf{S5}^t$ the following:

If $\vdash_{\mathbf{S5}^t_e} \Box^t(\varphi_1 \to \Box^t(\varphi_2 \to ... \to \Box^t(\varphi_{n-1} \to \Box^t\varphi_n)...))$,
and x is not free in $\varphi_1, ..., \varphi_{n-1}$, then
$\vdash_{\mathbf{S5}^t_e} \Box^t(\varphi_1 \to \Box^t(\varphi_2 \to ... \to \Box^t(\varphi_{n-1} \to \Box^t(\forall^e x)\varphi_n)...))$.

A completeness theorem can be shown for the actualist modal logic $\mathbf{S5}^t_e$.

Metatheorem: For each $\mathbf{S5}^t$-formula φ that is also an E-formula, $\vdash_{\mathbf{S5}^t_e} \varphi$ if, and only if, φ is **S5**-valid.

The Diodorean modalities, we have noted, are weaker than the Aristotelian modalities, and the corresponding quantified modal logic is not **S5** but the weaker system known as **S4.3**.

Definition: If L is a language, then φ is an **$\mathbf{S4.3}^t$-formula** of L if, and only if, φ belongs to every set K containing the atomic formulas of L and such that $\neg\varphi$, $\Diamond^f\varphi$, $(\varphi \to \psi)$, $(\forall x)\varphi$, $(\forall^e x)\varphi \in K$ whenever $\varphi, \psi \in K$ and x is a variable.

Definition: φ **is S4.3-valid** if, and only if, φ is an $\mathbf{S4.3}^t$-formula that is tense-logically true.

We obtain the system we call $\mathbf{S4.3}^t$ if we add to the axioms (A1)-(A10) of the logic of actual and possible objects all instances of schemas of the following forms:

$(S4.3^t\text{-}1)$	$\Box^f\varphi \to \varphi$
$(S4.3^t\text{-}2)$	$\Box^f(\varphi \to \psi) \to (\Box^f\varphi \to \Box^f\psi)$
$(S4.3^t\text{-}3)$	$\Box^f\varphi \to \Box^f\Box^f\varphi$
$(S4.3^t\text{-}4)$	$\Diamond^f\varphi \wedge \Diamond^f\psi \to \Diamond^f(\varphi \wedge \psi) \vee \Diamond^f(\varphi \wedge \Diamond^f\psi) \vee \Diamond^f(\psi \wedge \Diamond^f\varphi)$
$(S4.3^t\text{-}5)$	$\Diamond^f(x = y) \to \Box^f(x = y)$
$(S4.3^t\text{-}6)$	$(\forall x)\Box^f\varphi \to \Box^f(\forall x)\varphi$

As inference rules for $\mathbf{S4.3}^t$ we have the same inference as those for $\mathbf{S5}^t$, except except for having $\Box^f$ where $\Box^t$ occurs in those rules. It can be shown that for each $\mathbf{S4.3}^t$-formula φ, φ is a theorem of $\mathbf{S4.3}^t$ if, and only if, φ is **S4.3**-valid, which is our completeness theorem for $\mathbf{S4.3}^t$.

Metatheorem: For each $\mathbf{S4.3}^t$-formula φ, $\vdash_{\mathbf{S4.3}^t} \varphi$ if, and only if, φ is **S4.3**-valid.

For an actualist $\mathbf{S4.3}^t$ modal logic we need first to restrict the tensed $\mathbf{S4.3}^t$-formulas to tensed E-formulas. Then, to obtain the subsystem $\mathbf{S4.3}^t_e$ of $\mathbf{S4.3}^t$ when the latter is restricted to E-formulas, we must first delete the axiom schema ($\mathbf{S4.3}^t$-6), which is not an E-formula, and replace (A1)-(A10) of the logic of actual and possible objects by the axioms of the logic of actual objects *simpliciter*. We then adopt the same modal inference rules as already described for $\mathbf{S5}^t_e$, except for using $\Box^f$ instead of $\Box^t$ in those rules. Then, it can be shown that for each $\mathbf{S4.3}$-formula φ that is also an E-formula, φ is a theorem of $\mathbf{S4.3}^t_e$ if, and only if, φ is $\mathbf{S4.3}$-valid, which is our completeness theorem for $\mathbf{S4.3}^t$-formulas when the latter are restricted to E-formulas.

Metatheorem: For each $\mathbf{S4.3}^t$-formula φ that is also an E-formula, $\vdash_{S4.3^t_e} \varphi$ if, and only if, φ is $\mathbf{S4.3}$-valid.

Infinitely many other modal logics can be generated in ways similar to the above by various combination of tenses—e.g., merely iterating new occurrences of $\mathcal{F}$ in the definition of the Diodorean modalities will lead to new modalities. In addition to these temporal notions of modality, the semantics for yet another can be given corresponding roughly to the idea that a formula is conditionally necessary (in a given history at a given moment of that history) because of the way the past has been. The semantics for this notion also yields a completeness theorem for an **S5** type modal structure, and it may be used for a partial or full explication of the notions of causal modality and counterfactuals.

11 Causal Tenses in Relativity Theory

One of the defects of Aristotle's notion of necessity as a paradigm of a temporally-based modality is its exclusion of certain situations that are possible in special relativity theory as a result of the finite limiting velocity of causal influences, such as a light signal moving from one point of space-time to another. For example, relative to the present of a given local time T_0, a state of affairs can come to have been the case, according to special relativity, without its ever actually being the case.[18] That is, where $\mathcal{FP}\varphi$ represents φ's coming (in the future) to have been the case (in the past), and $\neg\Diamond^t\varphi$ represents φ's never actually being the case, the situation envisaged in special relativity might be thought to be represented by:

$$\mathcal{FP}\varphi \wedge \neg\Diamond^t\varphi. \qquad \textbf{(Rel)}$$

[18] Cf. Putnam, 1967.

This conjunction is incompatible with the connectedness (or linearity) assumption of the local time T_0 in question; for on the basis of the assumption of connectedness,

$$\mathcal{FP}\varphi \rightarrow \mathcal{P}\varphi \vee \varphi \vee \mathcal{F}\varphi$$

is tense-logically true, and therefore, by definition of $\Diamond^t$,

$$\mathcal{FP}\varphi \rightarrow \Diamond^t\varphi$$

is also tense-logically true. That is, $\mathcal{FP}\varphi$, the first conjunct of **(Rel)**, implies $\Diamond^t\varphi$, which contradicts the second conjunct of **(Rel)**, $\neg\Diamond^t\varphi$. The connectedness (or linearity) assumption cannot be given up, moreover, without violating the notion of a local time or of a world-line as an inertial reference frame upon which that local time is based. The notion of a local time is a fundamental construct not only of conceptualism and our common-sense framework but of natural science as well, as in the assumption of an *Eigenzeit* in relativity theory.

In conceptualism the connectedness of a local time is part of the notion of the self as a center of conceptual activity, and in fact it is one of the principles upon which the tense-logical cognitive schemata characterizing our conceptual orientation in time are constructed.

This is not to say that in the development of the concept of a self as a center of conceptual activity we do not ever come to conceive of the ordering of events from perspectives other than our own. Indeed, by a process that Jean Piaget calls *decentering*, children at the stage of concrete operational thought (7–11 years) develop the ability to conceive of projections from their own positions to that of others in their environment; and subsequently, by means of that ability, they are able to form operational concepts of space and time whose systematic coordination results essentially in the structure of projective geometry.[19]

Spatial considerations aside, however, and with respect to time alone, the cognitive schemata implicit in the ability to conceive of such projections can be represented in part by means of tense operators corresponding to those already representing the past and the future as viewed from one's own local time. That is, because the projections in question are to be based on actual causal connections between the momentary descriptive states of inertial frames, or world-lines, we can represent the cognitive schemata implicit in such projections by what we will here call *causal-tense operators*.

[19] Cf. Piaget 1972.

P_c : it causally was the case that ...
F_c : it causally will be the case that ...

Semantically, the causal-tense operators go beyond the standard tenses by requiring us to consider not just a single local time but a causally connected system of local times. The causal connections are between the momentary states of the different inertial reference frames, or continuants, upon which such local times are based; and, given the finite limiting velocity of light, these causal connections can be represented by **a signal relation** between the moments of the local times themselves—so long as we assume that the sets of moments of different local times are disjoint. (This assumption is harmless if we think of a moment of a local time as an ordered pair one constituent of which is the inertial frame upon which that local time is based.) The only constraint that should be imposed on such a signal relation is that it be a strict partial ordering, i.e., transitive and asymmetric.[20]

Thus, by **the causal past**, as represented by $\mathcal{P}_c$, we mean not just the past with respect to the here-now of our own local time, but also the past with respect to any momentary state of any other world-line that can send a signal to our here-now; and by **the causal future**, as represented by $\mathcal{F}_c$, we mean not just the future with respect to our here-now, but the future of any momentary state of any world-line to which we can send a signal from here-now. The geometric structure at a given momentary state of a world-line of a causally connected system is that of a Minkowski light-cone. That is, at each momentary state X of a world-line there is both **a prior light cone (the causal past)** consisting of all the momentary states (or space-time points) of world-lines that can send a signal to X and **a posterior light cone (causal future)** of all the momentary states (or space-time points) of world-lines that can receive a signal from X. Momentary states are then said to be **simultaneous** if no signal relation can be sent from one to the other.

> The *causal past* (prior light-cone) of the here-now momentary state X of a world-line (continuant) = the

[20] See Cocchiarella 1984, section 15, for the details of this semantics. The signal relation, incidentally, provides yet another example of a concrete interpretation of an accessibility relation between possible worlds, reconstrued now as the momentary states of the universe at different space-time points.

momentary states of world-lines that can send a signal to X.

The *causal future* (posterior light-cone) of the here-now momentary state X of a world line (continuant) = the momentary states of world-lines to which a signal can be sent from X.

A momentary state X of a world-line W_1 is *simultaneous* with a momentary state Y of a world-line W_2 if, and only if, no signal can be sent from X to Y, nor from Y to X.

Now because the signal relation has a finite limiting velocity, simultaneity will not be a transitive relation. As a result any one of a number of momentary states of one world-line can be simultaneous with the same momentary state of another world-line. This is what leads to the type of situation described by Putnam:

> Let Oscar be a person whose whole world-line is outside of the light-cone of me-now. Let me-future be a future 'stage' of me such that Oscar is in the lower half of the light cone of me-future [i.e., the prior cone of me-future]. Then, when that future becomes the present, it will be true to say that Oscar *existed*, although it will never have had such a truth value to say in the present tense 'Oscar exists now'. Things could come to *have been*, without its ever having been true that they *are*![21]

What all this indicates is that the possibility according to special relativity theory of a state of affairs coming to have been the case without its ever actually being the case is a possibility that should be represented in terms of the causal tense-operators $\mathcal{F}_c\varphi$ and $\mathcal{P}_c\varphi$—i.e., in terms of the causal past and causal future—and not in terms of the simple past- and future-tense operators $\mathcal{P}$ and $\mathcal{F}$, i.e., the past and future according to the ordering of events within a single local time.

Note that because there is a causal connection from the earlier to the later momentary states of the same local time, the signal relation is assumed to contain as a proper part the connected temporal ordering of the moments of

[21] Putnam 1967, p. 204.

each of the local times in such a causally connected system.[22] The following, in other words, are valid theses of such a causally connected system:

$$\mathcal{P}\varphi \rightarrow \mathcal{P}_c\varphi$$

$$\mathcal{F}\varphi \rightarrow \mathcal{F}_c\varphi$$

But note also that, because the signal relation has a finite limiting velocity, the converses of these theses will not also be valid in such a system. Were we to reject the assumption of relativity theory that there is a finite limit to causal influences, namely, the speed of light—as was implicit in classical physics and is still implicit in our commonsense framework where simultaneity is assumed to be absolute across space-time—then we would validate the converses of the above theses, in which case the causal tense operators would be completely redundant, which explains why they have no counterparts in natural language.

A related point is that unlike the cognitive schemata of the standard tense operators whose semantics is based on a single local time, those represented by the causal tense-operators are not such as must be present in one form or another in every speech or mental act. They are *derived* schemata, in other words, constructed on the basis of those decentering abilities whereby we are able to conceive of the ordering of events from a perspective other than our own. The importance and real significance of these derived schemata was unappreciated until the advent of special relativity.

One important consequence of the divergence of the causal from the standard tense operators is the invalidity of

$$\mathcal{F}_c\mathcal{P}_c\varphi \rightarrow \mathcal{P}_c\varphi \vee \varphi \vee \mathcal{F}_c\varphi$$

and therefore the consistency of

$$\mathcal{F}_c\mathcal{P}_c\varphi \wedge \neg\Diamond^t\varphi.$$

Unlike its earlier counterpart in terms of the standard tenses, this last formula is the appropriate representation of the possibility in special relativity of a state of affairs coming (in the causal future) to have been the case (in the causal past) without its ever actually being the case (in a given local time).

[22] See Carnap, 1958, Sections 49–50, for such an analysis of the notion of a causally connected system of local times.

Indeed, not only can this formula be true at some moment of a local time of a causally connected system, but so can the following formula[23]:

$$[\mathcal{P}_c\Diamond^t\varphi \vee \mathcal{F}_c\Diamond^t\varphi] \wedge \neg\Diamond^t\varphi.$$

Quantification over *realia*, which now includes things that exist in space-time with respect to any local time and not just with respect to a given local time, also finds justification in special relativity. For just as some states of affairs can come to have been the case in the causal past of the causal future without their actually ever being the case, so too there can be things that do not exist in the past, present or future of our own local time, but which nevertheless might exist in a causally connected local time at a moment that is simultaneous with our present. In this regard, reference to such objects as real even if not presently existing would seem hardly controversial—or at least not at that stage of conceptual development where our decentering abilities enable us to construct referential concepts that respect other points of view causally connected with our own. *Realia* encompass all the objects of temporal possibilism and possibly more as well.

Finally, it should be noted that there is also a causal counterpart to Diodorus's notion of possibility as what either is or will be the case, namely, possibility as what either is or causally will be the case:

$$\Diamond^{cf}\varphi =_{df} \varphi \vee \mathcal{F}_c\varphi.$$

Instead of the modal logic **S4.3**, this causal Diodorean notion of possibility results in the modal logic **S4**. Moreover, if we also assume, as is usual in special relativity, that the causal futures of any two moments t, t' of two local times of a causally connected system *eventually intersect*, i.e., that there is a moment w of a local time such that both t and t' can send a signal to w, then the thesis

$$\mathcal{F}_c\neg\mathcal{F}_c\neg\varphi \rightarrow \neg\mathcal{F}_c\neg\mathcal{F}_c\varphi$$

will be validated, and the causal Diodorean notion of possibility will then result in the modal system **S4.2**,[24] i.e., the system **S4** plus the thesis

$$\Diamond^{fc}\Box^{fc}\varphi \rightarrow \Box^{fc}\Diamond^{fc}\varphi.$$

[23] This formula would be true at a given moment t of a local time X if in either the prior cone or posterior cone of that moment there is a space-time point t' of a world-line Y such that φ is always true in Y, even though φ is never true in X. Putnam's Oscar example indicates how this is possible in relativity theory.

[24] Cf. Prior, 1967, p. 203.

Many other modal concepts can also be characterized in terms of a causally connected system of local times, including, e.g., the notion of something being necessary because of the way the past has been. What is distinctive about them all is the unproblematic sense in which they can be taken as material or metaphysical modalities.

12 Some Observations on Quantifiers in Tense and Modal Logic

Once the logic of actual and possible objects is extended by introduction of tense and modal operators, there are certain complications that arise in the application of Leibniz's law and the law of universal instantiation (and its dual, existential generalization) in tense and modal contexts. We describe some of these features here as well as certain laws regarding the commutation of tense and modal operators with quantifier phrases.

In describing some of the theorem schemas involving quantifiers, tenses and modal operators in these different logics, we shall use the following notation:

(t) $\vdash_t$: is a theorem of tense logic (of local time) with quantification over actual and
possible objects.

(t^e) $\vdash_{t^e}$: is a theorem of tense logic (of local time) with quantification over just actual objects.

(t^p) $\vdash_{t^p}$: is a theorem of tense logic (of local time) with quantification over just past and
present objects.

Note that what is provable in ($\mathbf{t}^e$) is provable in ($\mathbf{t}^p$), and what is provable in ($\mathbf{t}^p$) is provable in ($\mathbf{t}$). That is,

$$\{\varphi : \vdash_{t^e}\varphi\} \subseteq \{\varphi : \vdash_{t^p}\varphi\} \subseteq \{\varphi : \vdash_t\varphi\}$$

We may use $\vdash_{t^e}$, accordingly, to state what is provable in all three systems, and $\vdash_{t^p}$ for what is provable in ($\mathbf{t}^p$) and ($\mathbf{t}$). We will also use the following counterparts of notions already defined:

$$(\forall^f x)\varphi =_{df} (\forall x)[\mathcal{F}E!(x) \to \varphi]$$

$$\Diamond^p \varphi =_{df} (\varphi \vee \mathcal{P}\varphi)$$

$$\Box^p \varphi =_{df} \varphi \wedge \neg\mathcal{P}\neg\varphi$$

Leibniz's Law:

Assume that a, b are objectual constants, φ is a tensed formula, and ψ is obtained from φ by replacing one or more occurrences of a by occurrences of b. Then, we have the following theses about Leibniz's law:

(1) $\vdash_{t^e} \Box^t(a = b) \to (\varphi \leftrightarrow \psi)$

(2) $\vdash_{t^e} (x = y) \to (\varphi \leftrightarrow \psi)$, where x, y are variables.

(3) $\vdash_{t^e} (a = b) \to (\varphi \leftrightarrow \psi)$, if a does not occur in φ within the scope of a tense operator.

(4) $\vdash_{t^e} \Box^p(a = b) \to (\varphi \leftrightarrow \psi)$, if a does not occur in φ within the scope of a future tense operator.

(5) $\vdash_{t^e} \Box^f(a = b) \to (\varphi \leftrightarrow \psi)$, if a does not occur in φ within the scope of a past tense operator.

Identity and Non-identity:

Although identity and non-identity as expressed in terms of objectual variables is always necessary, that is,

$$\vdash_{t^e} (x = y) \to \Box^t(x = y)$$

$$\vdash_{t^e} (x \neq y) \to \Box^t(x \neq y),$$

The same theses are not true for objectual constants—unless they are "rigid designators," i.e., denote the same object at all times, which is symbolized as $(\exists x)\Box^t(a = x)$. Without assuming that, the relevant qualifications are as follows:

(6) $\vdash_t (\exists x)\Box^t(a = x) \wedge (\exists y)\Box^t(b = y) \to [a = b \leftrightarrow \Box^t(a = b)] \wedge$

$$[a \neq b \leftrightarrow \Box^t(a \neq b)]$$

$$\vdash_{t^e} (\exists^e x)\Box^t(a = x) \wedge (\exists^e y)\Box^t(b = y) \rightarrow [a = b \leftrightarrow \Box^t(a = b)] \wedge$$
$$[a \neq b \leftrightarrow \Box^t(a \neq b)]$$

$$\vdash_{t^p} (\exists^p_p x)\Box^t(a = x) \wedge (\exists^p_p y)\Box^t(b = y) \rightarrow [a = b \leftrightarrow \Box^t(a = b)] \wedge$$
$$[a \neq b \leftrightarrow \Box^t(a \neq b)]$$

Similar theorems hold when $\Box^t$ is uniformly replaced throughout **(6)** by $\Box^f$ and $\Box^p$, respectively.

Universal Instantiation:

The law of universal instantiation does not hold in general in these logics without qualification. The different qualifications are as follows, where x and y are variables, a is a term distinct from y, and a is free for x in φ:

(7) $\vdash_t (\exists y)\Box^t(a = y) \rightarrow [(\forall x)\varphi \rightarrow \varphi(a/y)]$

$\vdash_{t^e} (\exists^e y)\Box^t(a = y) \rightarrow [(\forall^e x)\varphi \rightarrow \varphi(a/y)]$

$\vdash_{t^p} (\exists^p_p y)\Box^t(a = y) \rightarrow [(\forall^p_p x)\varphi \rightarrow \varphi(a/y)]$

(8) If either a is a variable or x does not occur in φ within the scope of a past or future tense operator, then:

$\vdash_t (\forall x)\varphi \rightarrow \varphi(a/y)$

$\vdash_{t^e} (\exists^e y)(a = y) \rightarrow [(\forall^e x)\varphi \rightarrow \varphi(a/y)]$

$\vdash_{t^p} (\exists^p_p y)(a = y) \rightarrow [(\forall^p_p x)\varphi \rightarrow \varphi(a/y)]$

(9) If x does not occur in φ within the scope of a future tense operator, then:

$\vdash_{t^p} (\exists y)\Box^p(a = y) \rightarrow [(\forall x)\varphi \rightarrow \varphi(a/y)]$

$\vdash_{t^e} (\exists^e y)\Box^p(a = y) \rightarrow [(\forall^e x)\varphi \rightarrow \varphi(a/y)]$

$\vdash_{t^p} (\exists^p_p y)\Box^p(a = y) \to [(\forall^p_p x)\varphi \to \varphi(a/y)]$

(10) If x does not occur in φ within the scope of a past tense operator, then:

$\vdash_t (\exists y)\Box^f(a = y) \to [(\forall x)\varphi \to \varphi(a/y)]$

$\vdash_{t^e} (\exists^e y)\Box^f(a = y) \to [(\forall^e x)\varphi \to \varphi(a/y)]$

$\vdash_{t^p} (\exists^p_p y)\Box^f(a = y) \to [(\forall^p_p x)\varphi \to \varphi(a/y)]$

Laws of Commutation:

The possible quantifier $\exists$ commutes with both the past and future tense operators and therefore with $\Diamond^f$, $\Diamond^p$, and $\Diamond^t$ as well. Dually, $\forall$ commutes with $\neg\mathcal{P}\neg$ and $\neg\mathcal{F}\neg$ and therefore with $\Box^f$, $\Box^p$, and $\Box^t$ as well:

(11) $\vdash_t \mathcal{P}(\exists x)\varphi \leftrightarrow (\exists x)\mathcal{P}\varphi$ $\quad$ $\vdash_t \neg\mathcal{P}\neg(\forall x)\varphi \leftrightarrow (\forall x)\neg\mathcal{P}\neg\varphi$

$\vdash_t \mathcal{F}(\exists x)\varphi \leftrightarrow (\exists x)\mathcal{F}\varphi$ $\quad$ $\vdash_t \neg\mathcal{F}\neg(\forall x)\varphi \leftrightarrow (\forall x)\neg\mathcal{F}\neg\varphi$

$\vdash_t \Diamond^f(\exists x)\varphi \leftrightarrow (\exists x)\Diamond^f\varphi$ $\quad$ $\vdash_t \Box^f(\forall x)\varphi \leftrightarrow (\forall x)\Box^f\varphi$

$\vdash_t \Diamond^p(\exists x)\varphi \leftrightarrow (\exists x)\Diamond^p\varphi$ $\quad$ $\vdash_t \Box^p(\forall x)\varphi \leftrightarrow (\forall x)\Box^p\varphi$

$\vdash_t \Diamond^t(\exists x)\varphi \leftrightarrow (\exists x)\Diamond^t\varphi$ $\quad$ $\vdash_t \Box^t(\forall x)\varphi \leftrightarrow (\forall x)\Box^t\varphi$

The actual quantifier $\exists^e$ does not commute with the past or future tense operators except under special conditions, and even then different conditions are required for each direction—unless it is assumed that nothing ever comes to exist or ceases to exist, in symbols:

$\Box^t(\forall^e x)\Box^t E!(x)$ $\quad$ Nothing ever comes to exist or ceases to exist,

in which case $\exists^e$ commutes with $\Diamond^f$, $\Diamond^p$, and $\Diamond^t$ (and therefore, by duality, $\forall^e$ commutes with $\neg\mathcal{P}\neg$, $\neg\mathcal{F}\neg$, $\Box^f$, $\Box^p$, and $\Box^t$).

(12) $\vdash_{t^e} (\forall^e x)\neg\mathcal{P}\neg E!(x) \to [(\exists^e x)\mathcal{P}\varphi \to \mathcal{P}(\exists^e x)\varphi]$

$$\vdash_{t^e} \neg\mathcal{P}\neg(\forall^e x)\neg\mathcal{F}\neg E!(x) \rightarrow [\mathcal{P}(\exists^e x)\varphi \rightarrow (\exists^e x)\mathcal{P}\varphi]$$

$$\vdash_{t^e} (\forall^e x)\neg\mathcal{F}\neg E!(x) \rightarrow [(\exists^e x)\mathcal{F}\varphi \rightarrow \mathcal{F}(\exists^e x)\varphi]$$

$$\vdash_{t^e} \neg\mathcal{F}\neg(\forall^e x)\neg\mathcal{P}\neg E!(x) \rightarrow [\mathcal{F}(\exists^e x)\varphi \rightarrow (\exists^e x)\mathcal{F}\varphi]$$

$$\vdash_{t^e} \Box^t(\forall^e x)\Box^t E!(x) \rightarrow [(\exists^e x)\mathcal{P}\varphi \leftrightarrow (\exists^e x)\mathcal{P}\varphi]$$

$$\vdash_{t^e} \Box^t(\forall^e x)\Box^t E!(x) \rightarrow [(\exists^e x)\mathcal{F}\varphi \leftrightarrow \mathcal{F}(\exists^e x)\varphi]$$

$$\vdash_{t^e} \Box^t(\forall^e x)\Box^t E!(x) \rightarrow [(\exists^e x)\Diamond^f\varphi \leftrightarrow \Diamond^f(\exists^e x)\varphi]$$

$$\vdash_{t^e} \Box^t(\forall^e x)\Box^t E!(x) \rightarrow [(\exists^e x)\Diamond^p\varphi \leftrightarrow \Diamond^p(\exists^e x)\varphi]$$

$$\vdash_{t^e} \Box^t(\forall^e x)\Box^t E!(x) \rightarrow [(\exists^e x)\Diamond^t\varphi \leftrightarrow \Diamond^t(\exists^e x)\varphi]$$

Assumptions weaker than the condition that nothing ever comes into or goes out of existence—such as that everything presently existing always has existed and always will exist, or that everything now existing will never cease to exist, or that everything now existing always has existed yield commutations in only one direction.

$(\forall^e x)\Box^p E!(x)$ Everything presently existing always has existed

$(\forall^e x)\Box^f E!(x)$ Everything presently existing always will exist.

$(\forall^e x)\Box^t E!(x)$ Everything presently existing always has and always will exist.

$$\vdash_{t^e} (\forall^e x)\Box^t E!(x) \rightarrow [\Box^t(\forall^e x)\varphi \rightarrow (\forall^e x)\Box^t\varphi]$$

$$\vdash_{t^e} (\forall^e x)\Box^f E!(x) \rightarrow [\Box^f(\forall^e x)\varphi \rightarrow (\forall^e x)\Box^f\varphi]$$

$$\vdash_{t^e} (\forall^e x)\Box^p E!(x) \rightarrow [\Box^p(\forall^e x)\varphi \rightarrow (\forall^e x)\Box^p\varphi]$$

The quantifier $\exists^p_p$ commutes with the past and future tense operators in only one direction, each the converse to the other, and therefore it commutes with $\Diamond^p$ and $\Diamond^f$ in only one direction as well. Similarly, $\forall^p_p$ commutes with $\neg\mathcal{P}\neg$ and $\neg\mathcal{F}\neg$, and therefore with $\Box^p$ and $\Box^f$ in only one direction:

(13) $\vdash_{t^p} \mathcal{P}(\exists^p_p x)\varphi \to (\exists^p_p x)\mathcal{P}\varphi$ $\qquad$ $\vdash_{t^p} (\forall^p_p x)\neg\mathcal{P}\neg\varphi \to \neg\mathcal{P}\neg(\forall^p_p x)\varphi$

$\vdash_{t^p} (\exists^p_p x)\mathcal{F}\varphi \to \mathcal{F}(\exists^p_p x)\varphi$ $\qquad$ $\vdash_{t^p} \neg\mathcal{F}\neg(\forall^p_p x)\varphi \to (\forall^p_p x)\neg\mathcal{F}\neg\varphi$

$\vdash_{t^p} (\exists^p_p x)\Diamond^f\varphi \to \Diamond^f(\exists^p_p x)\varphi$ $\qquad$ $\vdash_{t^p} \Box^f(\forall^p_p x)\varphi \to (\forall^p_p x)\Box^f\varphi$

$\vdash_{t^p} \Diamond^p(\exists^p_p x)\varphi \to (\exists^p_p x)\Diamond^p\varphi$ $\qquad$ $\vdash_{t^p} (\forall^p_p x)\Box^p\varphi \to \Box^p(\forall^p_p x)\varphi$

$\forall^p_p$ commutes with $\Box^p$ in both directions if every past and present object always was a past or present object:

$$\vdash_{t^p} (\forall^p_p x)\Box^p[E!(x) \vee \mathcal{P}E!(x)] \to [\Box^p(\forall^p_p x)\varphi \leftrightarrow (\forall^p_p x)\Box^p\varphi]$$

Strong conditions are needed in order to commute $\forall^p_p$ with $\Box^t$, and in fact only a very strong condition suffices for commutation in both directions:

$$\vdash_{t^p} (\forall^p_p x)\Box^t[E!(x) \vee \mathcal{P}E!(x)] \to [\Box^t(\forall^p_p x)\varphi \to (\forall^p_p x)\Box^t\varphi]$$

$$\vdash_{t^p} \Box^t(\forall^p_p x)\Box^t[E!(x) \vee \mathcal{P}E!(x)] \to [\Box^t(\forall^p_p x)\varphi \to (\forall^p_p x)\Box^t\varphi]$$

13 Concluding Remarks

Tense logic is not the only framework in which both the logic of actual and possible objects and the logic of actual objects *simpliciter* have natural applications and in which the differences between possibilism and actualism can be made perspicuous. There is also, for example, the logic of belief and knowledge and the differences between the possible quantifier and the actual quantifier binding variables occurring free within the scope of operators for propositional attitudes. Still, even these other frameworks must presuppose some account of the logic of tenses, in which case the differences between possibilism and actualism within tense logic becomes paradigmatic. Indeed, as we have indicated, this is certainly the case for the differences between possibilism and actualism in modal logic, since some of the very first modal concepts ever to be discussed in the history of philosophy have been modal concepts that can be analyzed in the framework of tense logic.

References

[1] 1958, Carnap, R., *Introduction to Symbolic Logic and Its Applications*, Dover Publications, NYC, 1958.

[2] 1966, Cocchiarella, N.B., *Tense Logic: A Study of Temporal Reference*, Ph.D. dissertation, UCLA.

[3] 1974, Cocchiarella, N.B, "La Semantica della Logica del Tempo," in *La Logica del Tempo*, Boringhieri, Torino, 1974: 318–347.

[4] 1984, Cocchiarella, N.B, "Philosophical Perspectives on Quantification in Tense and Modal Logic," in *Handbook of Philosophical Logic*, vol. 11, eds. Gabbay, D. and F. Guenthner, Reidel, Dordrecht (1984), pp. 309-53.

[5] 1973, Hintikka, J., T*ime and Necessity, Studies in Aristotle's Theory of Modality*, Oxford University Press, London.

[6] 1965, Kalish, D. and R.M. Montague, "On Tarski's Formalization of Predicate Logic with Identity," *Arch. für Math. Logik und Grundl.*, vol. 7 (1965): 61–79.

[7] 1972, Piaget, Jean, *The Principles of Genetic Epistemology*, Routledge & Kegan Paul, London.

[8] 1967, Prior, A.N., *Past, Present and Future*, Oxford University Press, Oxford, 1967.

[9] 1967, Putnam, H., "Time and Physical Geometry," *The Journal of Philosophy*, vol. LXIV, 8 (1967); reprinted in Punam's *Mathematics, Matter and Method, Philosophical Papers, vol. 1*, Cambridge University Press, 1975.

[10] 1976, Saarinen, E., "Backwards-looking Operators in Tense Logic and Natural Language," in J. Hintikka *et al* (eds), *Essays on Mathematical Logic*, Reidel, Dordrecht, 1976.

[11] 1973, Vlach, F., *'Now' and 'Then': A formal Study in the Logic of Tense Anaphora*, Ph.D. dissertation, UCLA.